PART I

General Nuclear Concepts

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SECTION I

Some Basic Nuclear Terms

1. Nuclide: any nuclear species of a given number of protons and neutrons.

   Nucleon: either neutron or proton.

   Mass Number = A ≡ number of nucleons = N+Z

   $$^{57}_{27}\mathrm{Co}$$

   Atomic Number = Z ≡ number of protons

2. Isotopes: same Z; different A

   $^{12}_{6}C$ $^{13}_{6}C$ $^{14}_{6}C$

3. Isobars: same A; different Z

   $^{140}_{55}Cs$ $^{140}_{56}Ba$ $^{140}_{57}La$ $^{140}_{58}Ce$

4. Isotones: different A; same N

   $^{14}_{6}C$ $^{15}_{7}N$ $^{16}_{8}O$

5. Atomic Mass Unit (amu): 1 amu is 1/12 the mass of $^{12}_{6}C$.

6. Masses of atomic particles

* M_p = 1.00783 amu = mass of the proton   *Mass of proton = 1.00728*

M_n = 1.00867 amu = mass of the neutron

M_e = 0.00054860 amu = mass of the electron

M_H = 1.00783 amu = mass of the hydrogen atom

7. Units of radioactivity

curie(ci) = 2.22×10^{12} dpm (disintegrations/min)

(Defined: 3.7E10 d/s)

millicurie(mci) = 10^{-3} ci

microcurie(μci) = 10^{-6} ci

SECTION II

Nuclear Mass-Energy Relationships

A. Balancing Nuclear Equations

$$^{139}_{57}La + ^{1}_{1}H \rightarrow ^{138}_{58}Ce + 2\,^{1}_{0}n \qquad (1)$$

$$^{252}_{98}Cf \rightarrow ^{248}_{96}Cm + ^{4}_{2}He \qquad (2)$$

$$^{14}_{6}C \rightarrow ^{14}_{7}N + ^{0}_{-1}\beta \qquad (3)$$

B. Conversion Factors

TABLE 1. Commonly used metric units: Systems

Physical Quantity	mKs	cgs
Force (F)	newton (nt)	dyne
Mass (m)	kilogram (Kg)	gram (g)
Length (d)	meter (m)	centimeter (cm)
Time (t)	seconds (sec)	seconds (sec)
Acceleration (a)	$\frac{\text{meters}}{\text{sec}^2}$	$\frac{\text{centimeters}}{\text{sec}^2}$
Work (W)	joule (J)	erg

From $F = m \times a$

$$nt \equiv \frac{Kg \cdot m}{sec^2} \qquad dyne = \frac{g \cdot cm}{sec^2}$$

$$nt \equiv 10^5 \text{ dynes}$$

From $W = F \times d$

$$J \equiv nt \cdot m \equiv \frac{Kgm^2}{sec^2} \qquad erg \equiv dyne \cdot cm \equiv \frac{g \cdot cm^2}{sec^2}$$

$$J \equiv 10^7 \text{ ergs}$$

1. Important energy conversion factors and definitions

NOTE: Energy and work are interchangeable

Electron Volt (eV): the energy acquired when an electron is accelerated through a potential difference of 1 volt.

Calorie (cal): the quantity of heat which must be added to 1 gram of water (1 atmosphere pressure) to change its temperature from 14.5 to 15.5°C.

$$1\ eV = 1.602 \times 10^{-19} J$$

$$= 1.602 \times 10^{-12} ergs$$

$$= 3.829 \times 10^{-26} cal$$

$$= 1.074 \times 10^{-9}\ amu$$

$10^9 eV = GeV$ $\qquad$ $10^6 eV = MeV$

a. An important energy conversion factor is

$$1\ \frac{eV}{molecule} = 23.06\ \frac{Kcal}{mole} \tag{4}$$

This is obtained by

$$1\ \frac{eV}{molecule} = \frac{3.829 \times 10^{-20} cal}{molecule} \times \frac{Kcal}{10^3 cal} \times \frac{6.023 \times 10^{23} molecules}{mole}$$

where $6.023 \times 10^{23}\ \frac{molecules}{mole}$ is Avogadro's number

b. Another frequently used conversion factor is

$$1\ amu = 931\ MeV \tag{5}$$

This is obtained by

$$1\ amu = \frac{1\ eV}{1.074 \times 10^{-9}} \times \frac{MeV}{10^6\ eV}$$

$$= 931\ MeV$$

C. Comparison of Energy in Chemical and Nuclear Reactions

1. $C_{(s)} + O_{2(g)} \rightarrow CO_{2(g)} + 94.03 \frac{Kcal}{mole}$ (6)

Using $E = \Delta M \cdot C^2$ and solving for the mass

$$\Delta M = \frac{E}{c^2} = \frac{94.30 \frac{Kcal}{mole}}{(3.0 \times 10^{10} \frac{cm}{sec})^2} \quad \text{where } c = 3.0 \times 10^{10} \text{ cm/sec}$$

$$= 94.03 \text{ Kcal/mole} \times \frac{\text{lev/molecule}}{23.06 \text{ Kcal/mole}} \times 1.602 \times 10^{-12} \text{ erg/ev}$$

$$\times 6.023 \times 10^{23} \frac{\text{molecules}}{\text{mole}} \times 9.0 \times 10^{20} \frac{cm^2}{sec^2}$$

Since erg = $g\ cm^2/sec^2$)

$$= 4.37 \times 10^{-9} [\frac{g \cdot cm^2/sec^2}{cm^2/sec^2}]/\text{mole}$$

$$= 4.37 \times 10^{-9} g \quad (10^{-3}\% \text{ per mole of reactant mass})$$ (7)

2. ${}^3_2He + {}^1_0n \rightarrow {}^4_2He + 4.7 \times 10^8$ Kcal/mole (8)

$$\Delta M = \frac{E}{C^2} = \frac{4.7 \times 10^8 \frac{Kcal}{mole}}{c^2} = 2.2 \times 10^{-2} g \quad \sim 0.5\% \text{ of reactants mass}$$ (9)

D. Calculation of Energy (Q) in Nuclear Reactions

Convention used: $Q = -\Delta E$

$$\Delta M = [\text{Sum of atomic masses of reactants}] - [\text{Sum of atomic masses of products}] \qquad (10)$$

Note: Atomic masses (amu) = mass of nucleus + electrons

$$Q = \Delta M(\text{amu}) \times 931\ (\text{MeV/amu}) = \text{MeV} \qquad (11)$$

Positive $Q \equiv$ exoergic (occurs spontaneously)

Negative $Q \equiv$ endoergic (requires energy to occur)

Ex. $${}^{3}_{3}\text{He} + {}^{1}_{0}\text{n} \rightarrow {}^{4}_{2}\text{He} + Q \qquad (12)$$

${}^{3}_{2}\text{He} = 3.0160$ amu

${}^{1}_{0}\text{n} = 1.0087$ amu

${}^{4}_{2}\text{He} = 4.0026$ amu

$\Delta M = [3.0160 \text{ amu} + 1.0087 \text{ amu}] - [4.0026 \text{ amu}]$

$= 0.221$ amu

$Q = 0.0221$ amu x 931 MeV/amu

$= 20.58$ MeV

E. Nuclear Binding Energy

Binding Energy (BE): the energy released in the formation of a nucleus from the appropriate numbers of neutrons and hydrogen atoms

Ex: Calculate the BE of ${}^{4}_{2}He$

$$2\,{}^{1}_{1}H + 2\,{}^{1}_{0}n \rightarrow {}^{4}_{2}He \quad (13)$$

${}^{1}_{1}H = 1.0078$ amu

$\Delta M = 2[1.0078 + 1.0087] - 4.0026$

$= 0.03040$ amu

BE = 931 MeV/amu x 0.0304 amu

$= 28.30$ MeV (14)

Binding energy per nucleon = Be/A

$$BE/A = \frac{25.30}{4} = 7.08 \text{ MeV/nucleon} \quad (15)$$

Often [illegible] to as separation [illegible]

1. Binding energy of last nucleon.

Ex. Calculate BE of last neutron and of last proton in ^{235}U.

Last neutron:

$$^{234}_{92}U + ^{1}_{0}n = ^{235}_{92}U + Q \qquad (16)$$

$$Q = M_{^{234}U} + M_n - M_{^{235}U} \times 931 \text{ MeV/amu}$$

$$= [234.04090 + 1.00867 - 235.04392]\text{amu} \times 931 \text{ MeV/amu}$$

BE last neutron = 5.26 MeV

Last proton:

$$^{234}_{91}Pa + ^{1}_{1}H = ^{235}_{92}U + Q \qquad (17)$$

$$Q = [M_{^{234}Pa} + M_{^{1}_{1}H} - M_{^{235}U}] \times 931 \text{ MeV/amu}$$

$$= [234.04330 + 1.00783 - 235.04392] \text{ amu} \times 93 \text{ MeV/amu}$$

B.E. last proton = 6.71 MeV.

2. BE of a deuteron

$${}^{1}_{1}H + {}^{1}_{0}n \rightarrow {}^{2}_{1}H \qquad (18)$$

BE = 2.2 MeV
BE/nucleon = 1.1 MeV

3. Binding energy per nucleon, BE/A

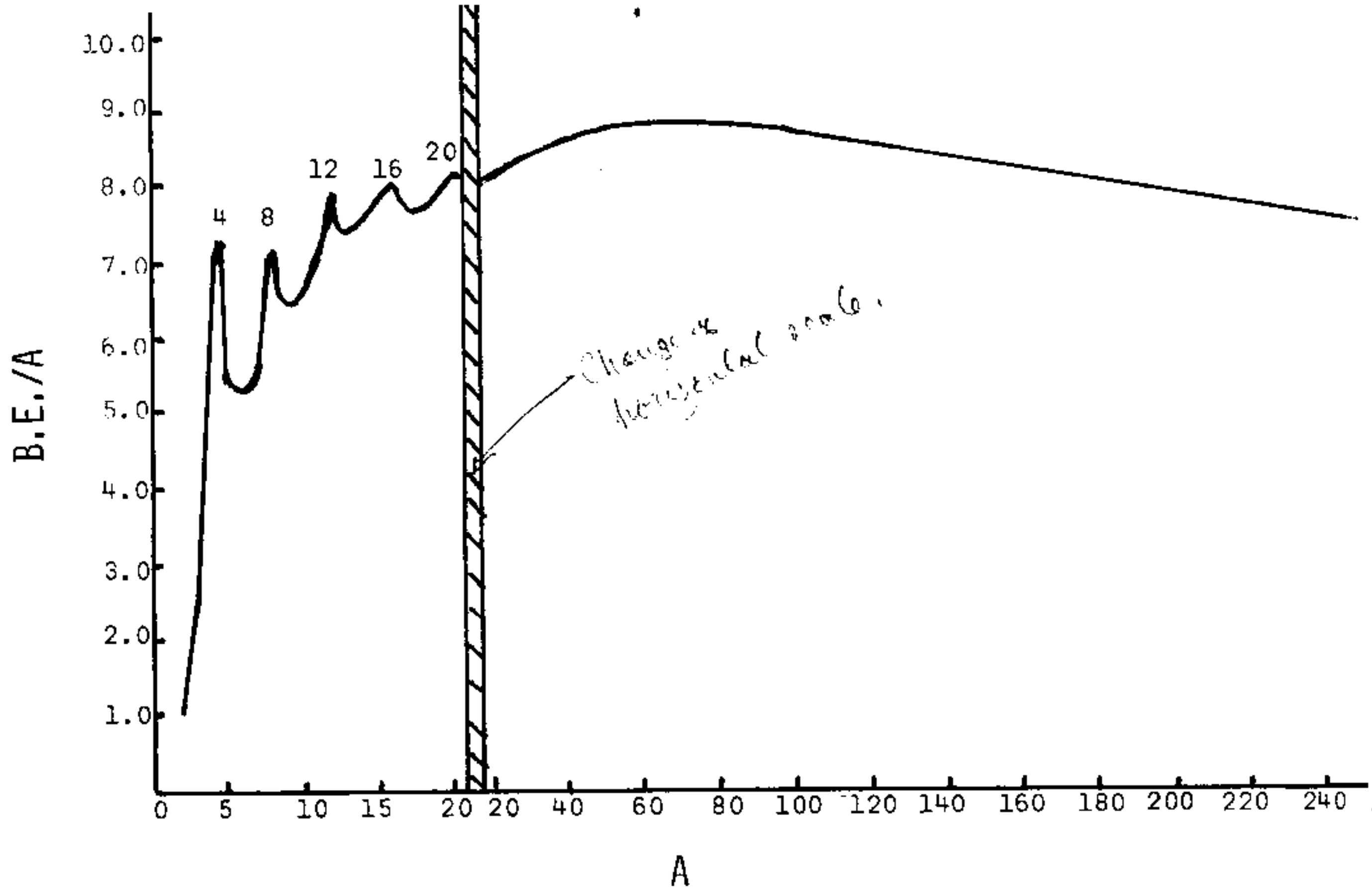


Fig. 1 Plot of the binding energy per nucleon (MeV/nucleon) as a function of the mass number.

Note: BE/A ∿ 7-9 MeV

Magic Numbers - 2, 8, 20, 28, 50, 82, and 126.

Exoergic reactions:

a) <u>fusion</u> of light elements (A<40)

b) <u>fission</u> of heavy elements (A>100).

Discussion of magic numbers on the tape is confusing, since it does not distinguish clearly between proton number, neutron number, and mass number. The magic numbers refer to either proton number or neutron number, whereas Fig 1 is plotted as a function of mass number. Thus the peak of A=4 corresponds to the doubly magic nuclide with Z=N=2.

SECTION III

Nuclear Radius

For a sphere of incompressible particles

$$V \propto A$$

For a sphere, $V \propto R^3$

$$R \propto A^{1/3}$$

$$R = R_0 A^{1/3}; \quad R_0 = 1.4 \times 10^{-13} \text{ cm.}$$

Defining 10^{-13} cm as one femtometer (fm),

$$R = 1.4 A^{1/3} \text{ fm} \qquad (19)$$

Examples: 1. For ^{238}U $\quad R = 1.4(238)^{1/3}\text{fm} = 8.7\text{fm}$

2. For ^{80}Br $\quad R = 1.4(80)^{1/3}\text{fm} = 6.0\text{fm}$

SECTION IV

Nuclear Coulomb Barrier

$$V_c(\text{ergs}) = \frac{(Ze)_1(Ze)_2}{D(\text{cm})}$$

e = electrical unit charge

$= 4.8 \times 10^{-10}$ esu

$$D = R_0 \times 10^{-13}(A_1^{1/3} + A_2^{1/3}) \text{ (cm)} \qquad (20)$$

$$\therefore V_c(\text{ergs}) = \frac{Z_1 Z_2 e^2}{R_0 \times 10^{-13}(A_1^{1/3} + A_2^{1/3})}$$

$$V_c(\text{MeV}) = V_c(\text{ergs}) \times 6.24 \times 10^5 \frac{\text{MeV}}{\text{ergs}}$$

or

1.43??? = 1.440

$$V_c(\text{MeV}) = \frac{1.439\ Z_1 Z_2}{R_0(A_1^{1/3} + A_2^{1/3})} \qquad (21)$$

when R_0 is given in fm

Example: Calculate the coulomb barrier in MeV for the reaction of ${}^{238}_{92}U + {}^{4}_{2}He$ R_o=1.5 fm

(R_o=1.4 fm)

$$V_c(\text{MeV}) = \frac{1.439 \times 92 \times 2}{1.4(238^{1/3} + 4^{1/3})} = 24.2 \text{ MeV} \qquad (22)$$

Note: No coulomb barrier for a neutron.

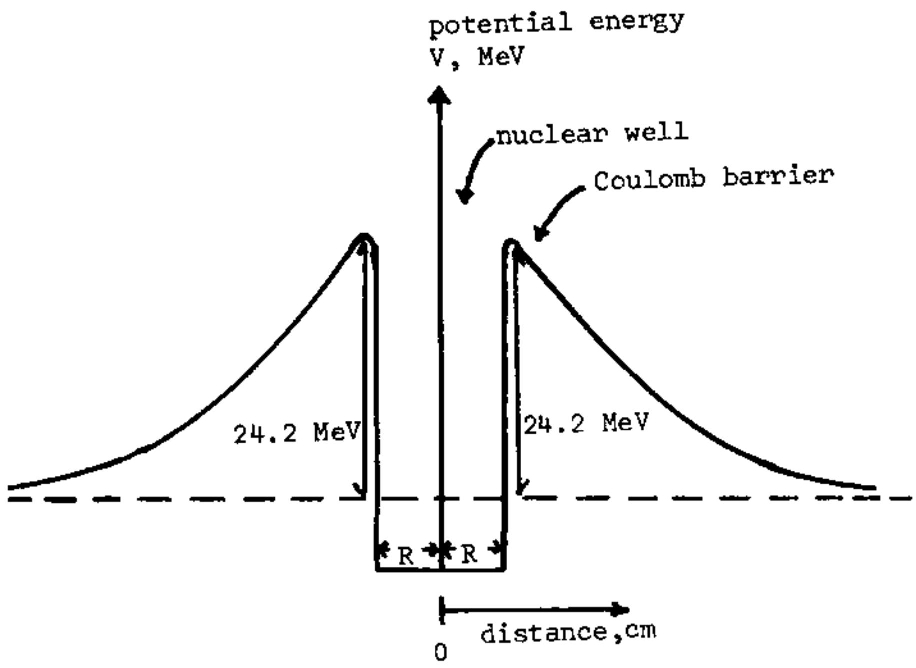


Fig. 2 Nuclear coulomb barrier and potential well for $^{238}_{92}U$ and $^{4}_{2}He$.

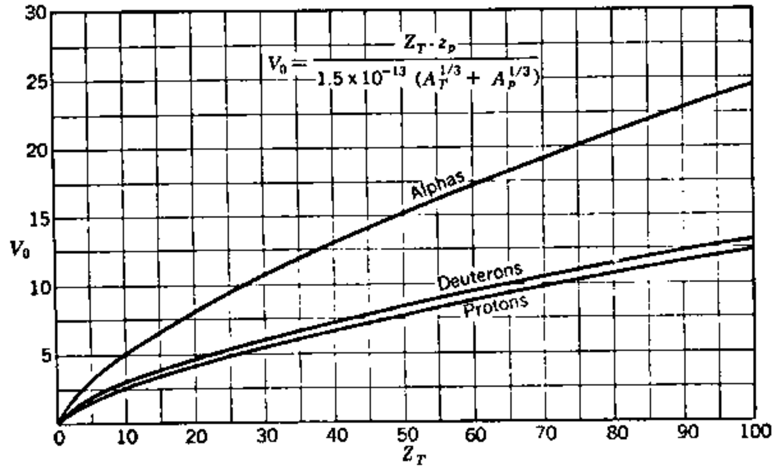
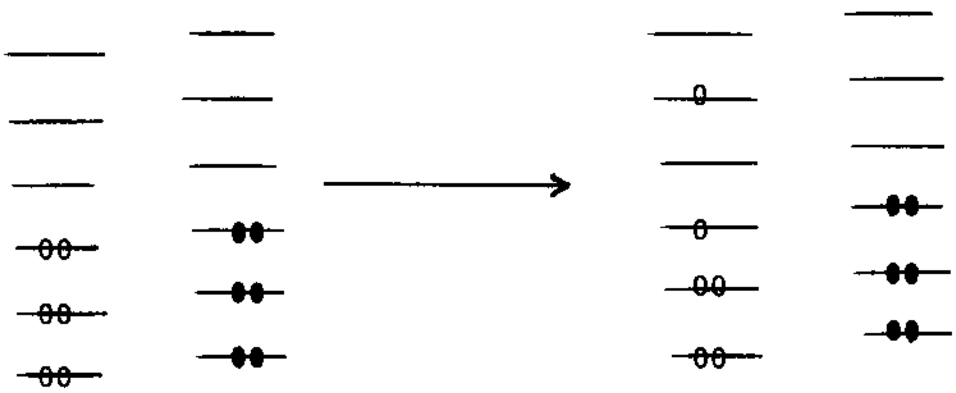


Fig. 3. The Coulomb barrier height as a function of Z_T, the atomic number of the target.

SECTION V

Nuclear Models

A. The Shell Model



Neutron Levels

Proton Levels

Excited nucleus in which a neutron is in a higher level

Fig. 4. Process of nuclear excitation by promotion of a neutron) in the shell model. The nucleus returns to the ground state by emission of one or more gamma rays.

B. The Liquid-Drop Model

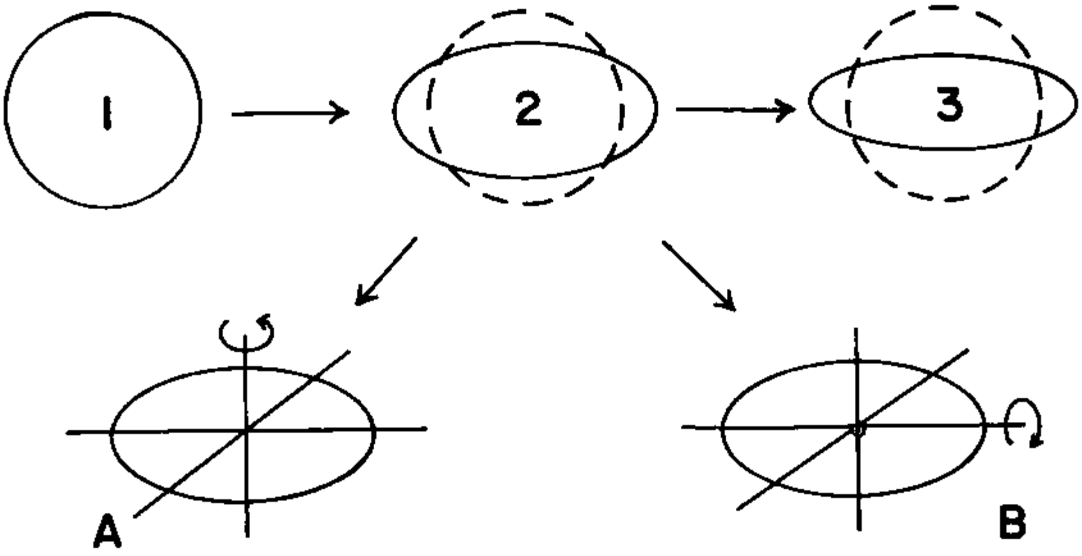


FIG. 5 Excitation of the nucleus into different vibrational (2 and 3) and rotational (A and B) states according to the liquid drop model.

C. The Unified Model

1. Combination of shell and liquid drop models.

2. As Z or N values approach "magic numbers", the nuclear liquid droplet becomes more spherical and "stiffer" (requires more energy to deform).

3. For "stiff" nuclei, individual nucleon excitations require less energy than the rotational and vibrational excitations.

SECTION VI

Problems

1. When the copper-63 nuclide is bombarded with deuterons, six different transmutations may occur. Complete the equations for these:

   a. $^{63}_{29}Cu + ^{2}_{1}H \longrightarrow$ _____ $+ ^{1}_{0}n$

   b. $^{63}_{29}Cu + ^{2}_{1}H \longrightarrow ^{64}_{29}Cu +$ _____

   c. $^{63}_{29}Cu + ^{2}_{1}H \longrightarrow ^{63}_{30}Zn +$ _____

   d. $^{63}_{29}Cu + ^{2}_{1}H \longrightarrow$ _____ $+ ^{4}_{2}He$

   e. $^{63}_{29}Cu + ^{2}_{1}H \longrightarrow ^{62}_{29}Cu +$ _____

   f. $^{63}_{29}Cu + ^{2}_{1}H \longrightarrow ^{65}_{30}Zn +$ _____

2. Calculate Q for the following reactions.

   a. $^{208}_{82}Pb + ^{1}_{1}H \longrightarrow ^{207}_{83}Bi + 2^{1}_{0}n + Q$

   b. $^{102}_{45}Rh + ^{4}_{2}He \longrightarrow ^{103}_{47}Ag + 3^{1}_{0}n + Q$

c. $^{27}_{13}Al + ^{1}_{0}n \longrightarrow ^{28}_{13}Al + Q$

d. $^{79}_{35}Br + ^{1}_{0}n \longrightarrow ^{76}_{33}As + ^{4}_{2}He + Q$

Atomic Masses (amu)

$^{1}_{1}H$ = 1.0078	$^{207}_{83}Bi$ = 206.9784	$^{79}_{35}Br$ = 78.9183
$^{4}_{2}He$ = 4.0026	^{102}Rh = 101.9068	$^{76}_{33}As$ = 75.9242
$^{1}_{0}n$ = 1.0087	$^{103}_{47}Ag$ = 102.9083	
γ ray = no rest mass	$^{27}_{13}Al$ = 26.9815	
$^{208}_{82}Pb$ = 207.9766	$^{28}_{13}Al$ = 27.9819	

3. Calculate the total binding energy and the binding energy per nucleon for:

	Masses (amu)
a. $^{24}_{12}Mg$	$^{24}_{12}Mg$ = 23.9850
b. $^{60}_{27}Co$	$^{60}_{27}Co$ = 59.9338
c. $^{197}_{79}Au$	$^{197}_{79}Au$ = 196.9665

4. Calculate the radii of the following nuclei using R_0 = 1.4×10^{-13} cm.

^{4}He, ^{16}O, ^{56}Fe, ^{75}As, ^{112}Cd, ^{165}Ho, ^{206}Pb, ^{238}U, and ^{256}Fm.

5. Plot the radius values calculated in the preceding problem as a function of mass number and discuss the the resulting curve.

6. Calculate the coulomb barrier of a) ^{238}U, and, b) ^{56}Fe nucleus to a proton when they are just in contact.

7. The density of metallic aluminum is 2.7 g/cm^3. Calculate the density of the ^{27}Al nucleus, using R_0 = 1.4 fm and $M_{^{27}Al}$ = 27.0 amu, and compare the result with the 2.7 g/cm^3 for the atom in the metallic lattice.