

PART II

Nuclear Stability and Radioactivity

Outline

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 - B. Nuclear Forces
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SECTION I

Stability Patterns

<u>Abundance</u>	<u>Proton No.</u>	<u>Neutron No.</u>	<u>Mass Number</u>
60%	even	even	even
40%	even odd	odd even	odd odd
4 Nuclei:	odd	odd	even
(${}^2_1\text{H}$, ${}^6_3\text{Li}$, ${}^{10}_5\text{B}$, ${}^{14}_7\text{N}$)			

A. Odd-Even Rules for Nuclear Stability

1. Elements of odd Z have at most 2 stable isotopes.
2. Adjacent isobars, ${}^A_Z\text{M}$ and ${}^A_{Z+1}\text{M}$, cannot both be stable.

Ex: Technetium (Z=43) - no stable isotopes.

Stable Values of A.

${}^{42}\text{Mo}$	92	94	95	96	97	98	100
${}^{43}\text{Tc}$					[97]	[99]	
${}^{44}\text{Ru}$				96		98	99 100 101 102 104

[] indicates values of A most likely to be stable for Z=43.

3. Isobars separated by 2 units of Z can be stable

e.g. ${}_{28}^{64}\text{Ni}^{36}$ and ${}_{30}^{64}\text{Zn}^{34}$ stable but ${}_{29}^{64}\text{Cu}^{35}$ radioactive

(neutron numbers are superscripts on right).

B. Nuclear forces

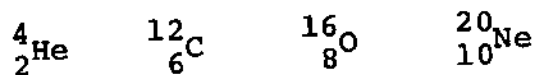
1. Coulomb electrostatic force: exists between protons; for nucleus it is repulsive, charge dependent, long range, proportional to Z^2/D (D = distance of proton-proton separation).

2. Nuclear force: exists between all nucleons and is attractive, charge independent, short range.

C. Neutron to Proton Ratio

1. Light nuclei

For stability $N = Z$; e.g.



2. As number of protons increase, in heavier nuclei when $Z=N$, p-p repulsion tends to outweigh sum of p-p, p-n, and n-n attractions. Thus, number of neutrons must increase at a more rapid rate than

number of protons. As Z increases, $n-p$ must increase (e.g.: $n/p = 1.6$ for uranium).

3. n/p for stability usually has a narrow range of values for a particular element.

		A of Stable Isotopes	n/p
even Z	50^{Sn}	112,114,115,116,117,118,119, 120,124	1.24-1.40
odd Z	49^{In}	113	1.31
	51^{Sb}	121,123	1.37-1.41

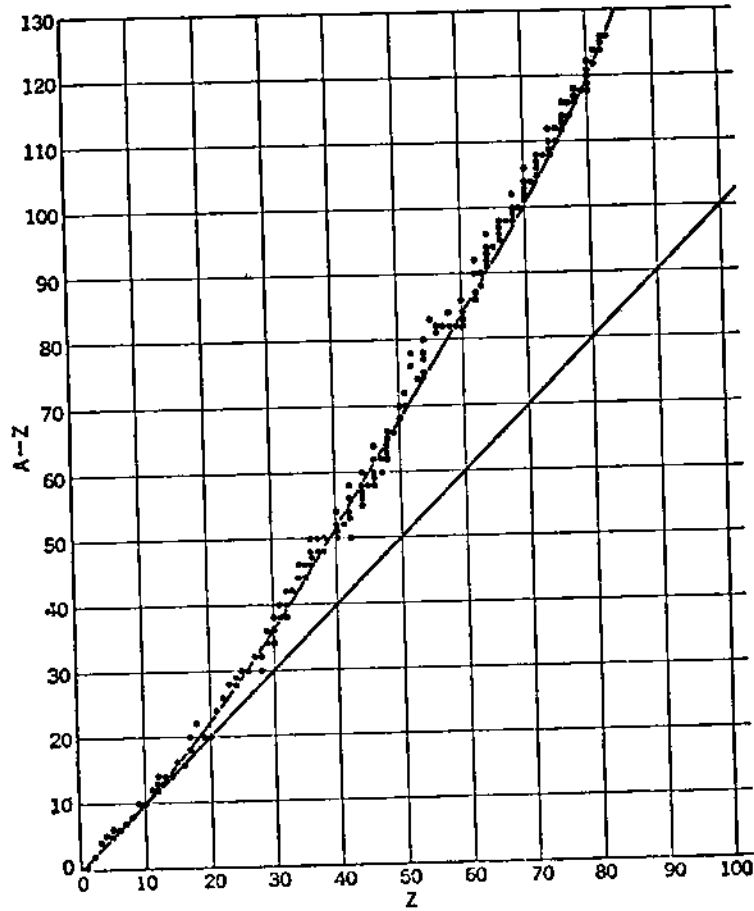


FIG. 1 Plot of the number of neutrons ($A-Z$) vs. the number of protons Z for stable nuclei ($>10\%$ abundance).

SECTION II

Modes of Radioactive Decay

3 modes of radioactive decay

1. Alpha (α) decay
2. Beta (β^- or β^+) decay and electron capture
3. Gamma (γ) decay

Spont. and Fission

A. Rules for Calculation of Nuclear Decay Energies

The Q value, or the energy released may be calculated from the atomic masses if they are known.

$$\text{Recall: } Q \equiv [\text{Sum of the atomic masses of the reactants}] \\ - [\text{Sum of the atomic masses of the products}]$$

$$Q \equiv \Delta M \text{ amu} \times 931 \text{ MeV/amu}$$

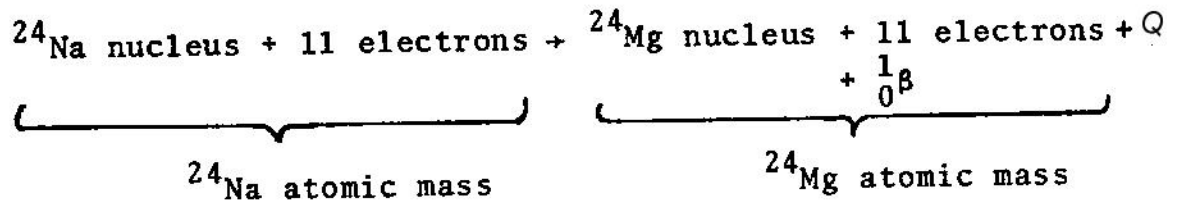
If Q is positive \equiv exoergic (occurs spontaneously)

negative \equiv endoergic (requires energy to occur).

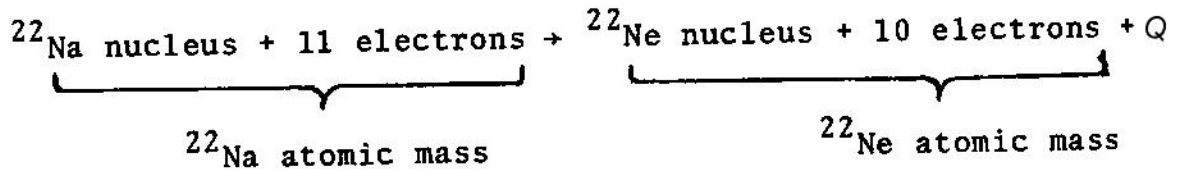
1. In equations involving negatron (β^-), electron capture (EC) or alpha (α) decay, it is not necessary to add or subtract electron masses.

Examples:

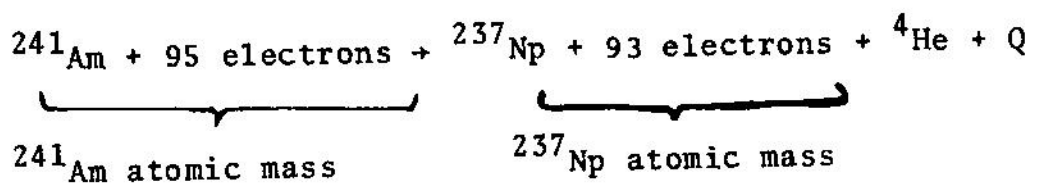
a) β^- Decay



b) EC Decay



c) Alpha Decay



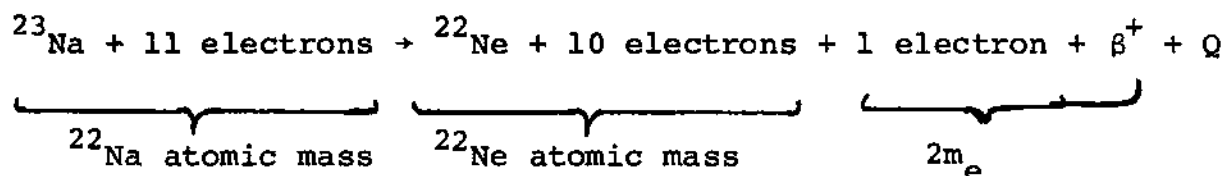
2. In equations involving positron (β^+) emission, it is necessary to subtract an additional two electron masses (i.e. 1.02 MeV) from the difference in masses of the initial and final product nuclei to obtain Q

$$Q = [\text{Sum of atomic masses of the reactants}] - [\text{Sum of atomic masses of the products}] - 1.02 \text{ MeV}$$

$$Q = \Delta m \text{ amu} \times 931 \text{ MeV/amu} - 1.02 \text{ MeV}$$

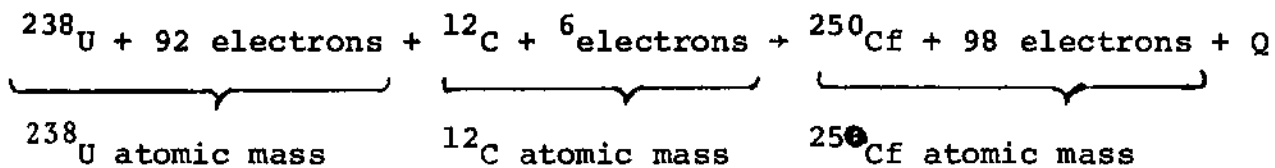
Example

β^+ Decay



3. It is not necessary to subtract electron masses in equations of nuclear reactions induced by charged particles or neutrons

Example



B. Alpha (α) Decay (Heavier Elements)



$$\Delta M = 238.0508 - 4.0026 - 234.0436$$

$$= + 0.0046 \text{ amu}$$

$$Q = (931 \text{ MeV/amu}) (0.0046 \text{ amu}) = 4.28 \text{ MeV}$$

Energy of the α particle

$$E_{\alpha} = \frac{M_{\text{Th}}}{M_{\text{Th}} + M_{\text{He}}} \times Q \approx \frac{A_{\text{Th}}}{A_{\text{Th}} + A_{\text{He}}} \times Q$$

$$= \frac{234}{238} \times 4.28 \text{ MeV} = 4.21 \text{ MeV} \quad (2)$$

Note: α -decay energy is discrete as transitions occur between states of definite energy.

$$E_{\text{recoil}} = E_{\alpha} \left(\frac{4}{238}\right) = 0.07 \text{ MeV} = 70 \text{ keV} \quad \text{carried off} \quad (3)$$

by residual
 ${}_{90}^{234}\text{Th}$ nucleus

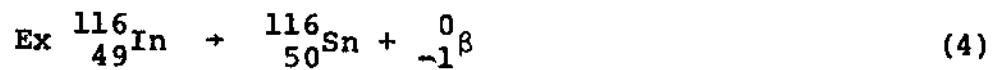
Note: $Q = E_{\alpha} + E_{\text{recoil}}$

$$= 4.21 \text{ MeV} + 0.07 \text{ MeV}$$
$$= 4.28 \text{ MeV}$$

C. Beta Decay

1. Negatron (β^-) Decay

- a. Occurs throughout the periodic table.
- b. Neutron rich nuclides attempt to achieve a stable n/p ratio by β^- decay.



$$Q_{\beta^-} = 3.35 \text{ MeV}$$

$$E_{\text{recoil}} = \frac{M_e}{M_{e^-} + M_{\text{Sn}}} \cdot Q_{\beta^-} = 5.49 \times 10^{-6} \text{ amu} \cdot Q_{\beta^-}$$

$$= 1.59 \times 10^{-5} \text{ MeV} \approx 16 \text{ eV carried off by residual } {}^{116}\text{Sn} \text{ nucleus.} \quad (5)$$

- c. Energy of emitted β^- particles from 0 to Q_{β^-}
(average energy $\approx 1/3 Q_{\beta^-}$)

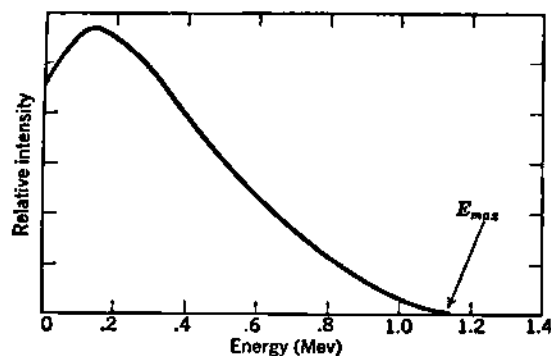
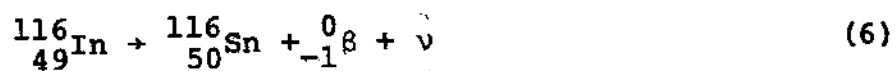


Fig. 2. Energy Spectrum for β^- rays

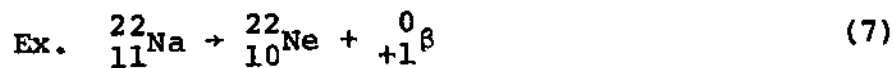
- d. Remaining energy carried off by neutrino,
(uncharged particle of zero rest mass).

Complete reaction



2. Positron (β^+) Decay

- a. Occurs if n/p ratio is too low



$$Q_{\beta^+} = 1.77 \text{ MeV}$$

- b. Energy of emitted positron averages about $0.4 Q_{\beta^+}$

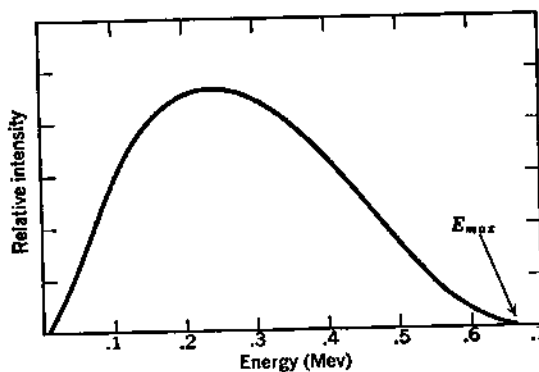


Fig. 3. Energy spectrum for β^+ rays

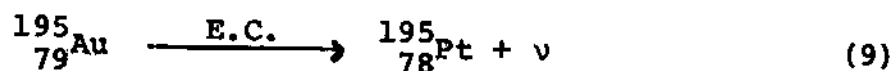
- c. Remaining energy carried off by neutrino (ν).

Complete reaction



3. Electron Capture

- a. Occurs when n/p ratio is too low
- b. Electron is absorbed from extranuclear electron shells of the atom (predominantly, the K shell)



$$\Delta m = 0.000283 \text{ amu} \times 931 \text{ MeV/amu} = 0.263 \text{ MeV}$$

$$Q_{\text{EC}} = 0.263 \text{ MeV}$$

- c. E.C. vs β^+

If the available decay energy [i.e. $\Delta m(\text{amu}) \times 931$ (MeV/amu)] is less than 1.02 MeV, positron emission is impossible and decay must take place by electron capture, E.C. If the available decay energy is greater than or equal to 1.02 MeV, decay can occur by either β^+ or EC or both.



$$M_{{}^{22}_{\text{Na}}} = 21.99444 \text{ amu}$$

$$M_{{}^{22}_{\text{Ne}}} = 21.99139 \text{ amu}$$

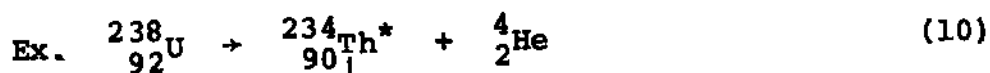
$$\text{Available energy} = (3.052 \times 10^{-3} \text{ amu}) (931 \text{ MeV/amu})$$

$$= 2.94 \text{ MeV}$$

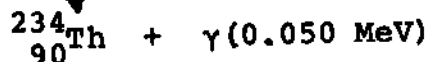
This reaction can go either by β^+ or EC or both. In reality ${}^{22}_{\text{Na}}$ decays 79% by β^+ and 11% by EC.

D. Gamma (γ) Decay

1. Must accompany another form of radioactive decay or occur as a consequence of a nuclear reaction which leaves a nucleus in an excited state.
e.g., follows α decay



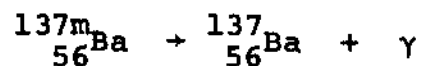
*=excited state



2. Metastable states occur when the excited state lasts a measurable amount of time.



m = metastable



3. Instead of γ -ray emission, orbital electron can be ejected; called a conversion electron.

$$E_e = E_\gamma - E_{BE}$$

E_e - energy of conversion electron

E_γ - energy of γ ray if it were to be emitted

E_{BE} - binding energy of orbital electron (conversion electron)

NOTE: In the examples given for α and β decay the energy of the recoiling nucleus was calculated. This energy imparted to the residual nucleus by the emission of a particle may be sufficient to rupture some or all of the chemical bonds holding the atom in a molecule.

The recoil energy in gamma decay is insufficient to cause bond rupture but such decay is associated with ionization of extranuclear electrons. This can result in bond rupture. The study of these processes are contained in a branch of nuclear sciences called "hot-atom" chemistry.

SECTION III
Decay Schemes

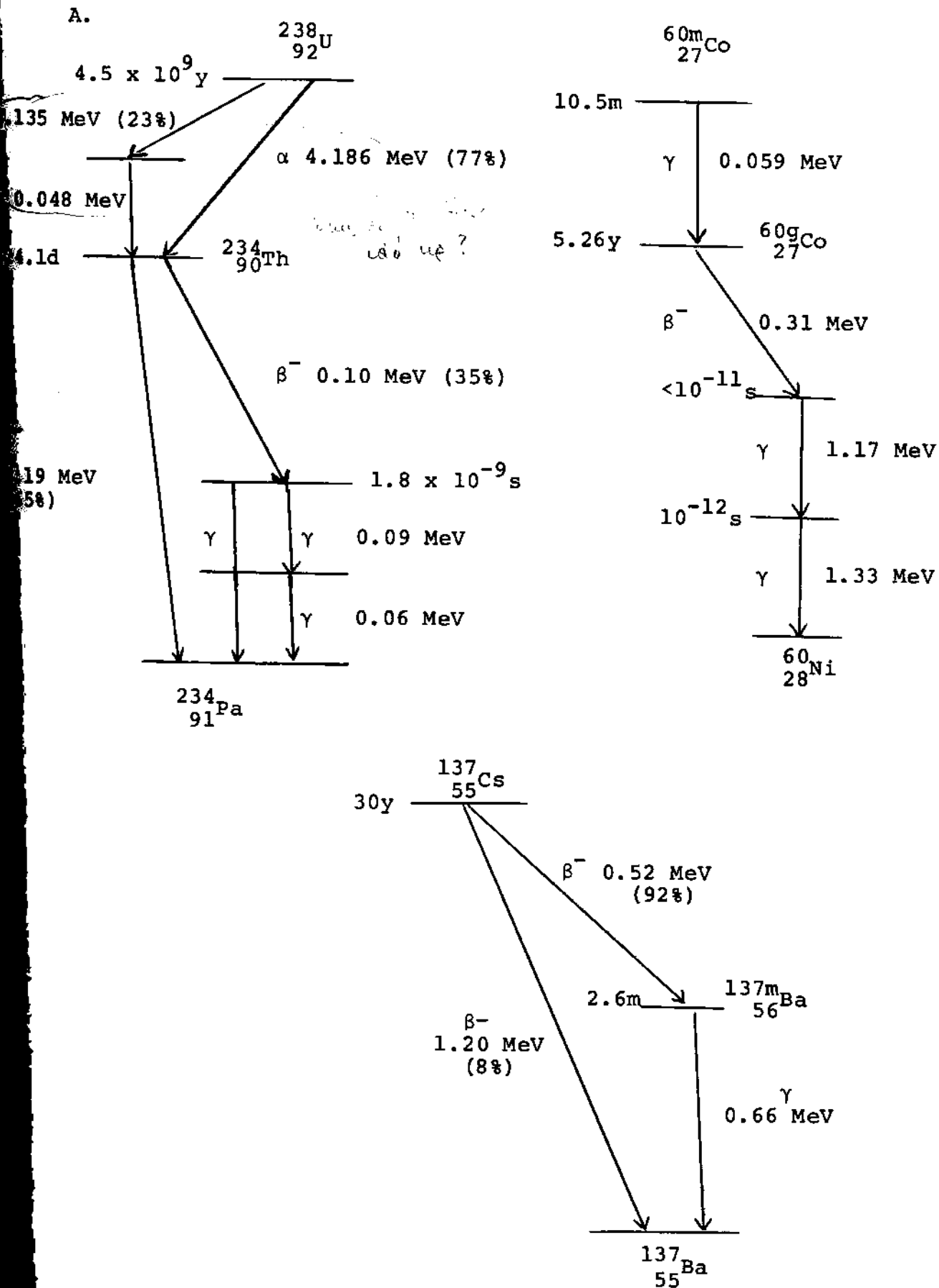


FIG. 4. Decay Schemes for $^{238}_{92}\text{U}$, $^{60\text{m}}_{27}\text{Co}$ and $^{137}_{55}\text{Cs}$

SECTION IV

Radioactive Decay Law

A. The Decay Constant and Half Life

A(disintegration/min) \propto N(number of atoms)

$$A = \lambda N \quad (13)$$

λ = decay constant

$$-dN/dt = \lambda N$$

$$N_t = N_0 e^{-\lambda t}$$

$$\log_{10} N_t = \log_{10} N_0 - \frac{\lambda t}{2.3}$$

$$A_t = A_0 e^{-\lambda t} \quad (13a)$$

$$\log_{10} A_t = \log_{10} A_0 - \frac{\lambda t}{2.3} \quad (13b)$$

When $A_t = 0.5 A_0$, $t = t_{1/2}$, the half-life.

$$\frac{A_t}{A_0} = 0.5 = e^{-t_{1/2} \lambda}$$

$$-\lambda t_{1/2} = \log_e 0.5$$

$$t_{1/2} = \frac{-\log_e 0.5}{\lambda}$$

$$t_{1/2} = \frac{0.693}{\lambda} \quad (14)$$

The decay constant (and half-life) is a definite characteristic of the nucleus. The half-life is the time required for 1/2 of a statistically large number of radioactive nuclei to undergo decay.

- B. Specific Activity (SA) - total radioactivity of a given isotope per gram of the radioactive isotope (can also be per gram of a compound or per gram of the element).

$$SA \left(\frac{\text{dpm}}{\text{g}} \right) = \frac{0.693 N}{T_{1/2}} = \frac{0.693}{T_{1/2}} \times \frac{6.023 \times 10^{23} \text{ atoms/mole}}{\text{atomic weight grams/mole}} \quad (15)$$

where $T_{1/2}$ = half-life in minutes

N = number of radioactive atoms/gram

$$= \frac{6.023 \times 10^{23} \text{ atoms/mole}}{\text{atomic weight gram/mole}}$$

Note: atomic weight can again refer to the isotope, compound, or element

or

$$SA\left(\frac{ci}{g}\right) = SA\left(\frac{dpm}{g}\right) \times \frac{\text{curie}}{2.22 \times 10^{12} \text{dpm}} \quad (16)$$

Ex. Calculate the SA in $\frac{dpm}{gm}$ and $\frac{ci}{gm}$ of ^{22}Na ($T_{1/2}=2.6y$).

(per gram of the radioisotope)

$$T_{1/2} = 2.6y = 1.367 \times 10^6 \text{m}$$

$$\begin{aligned} SA\left(\frac{dpm}{g}\right) &= \frac{0.693}{1.367 \times 10^6 \text{m}} \times \frac{6.023 \times 10^{23} \text{ atoms/mole}}{22 \text{ g/mole}} \\ &= 1.39 \times 10^{16} \frac{dpm}{g} \end{aligned}$$

$$SA\left(\frac{ci}{g}\right) = \frac{1.39 \times 10^{16} \frac{dpm}{g}}{2.22 \times 10^{12} \text{c}} = 6.252 \times 10^3 \text{ci}$$

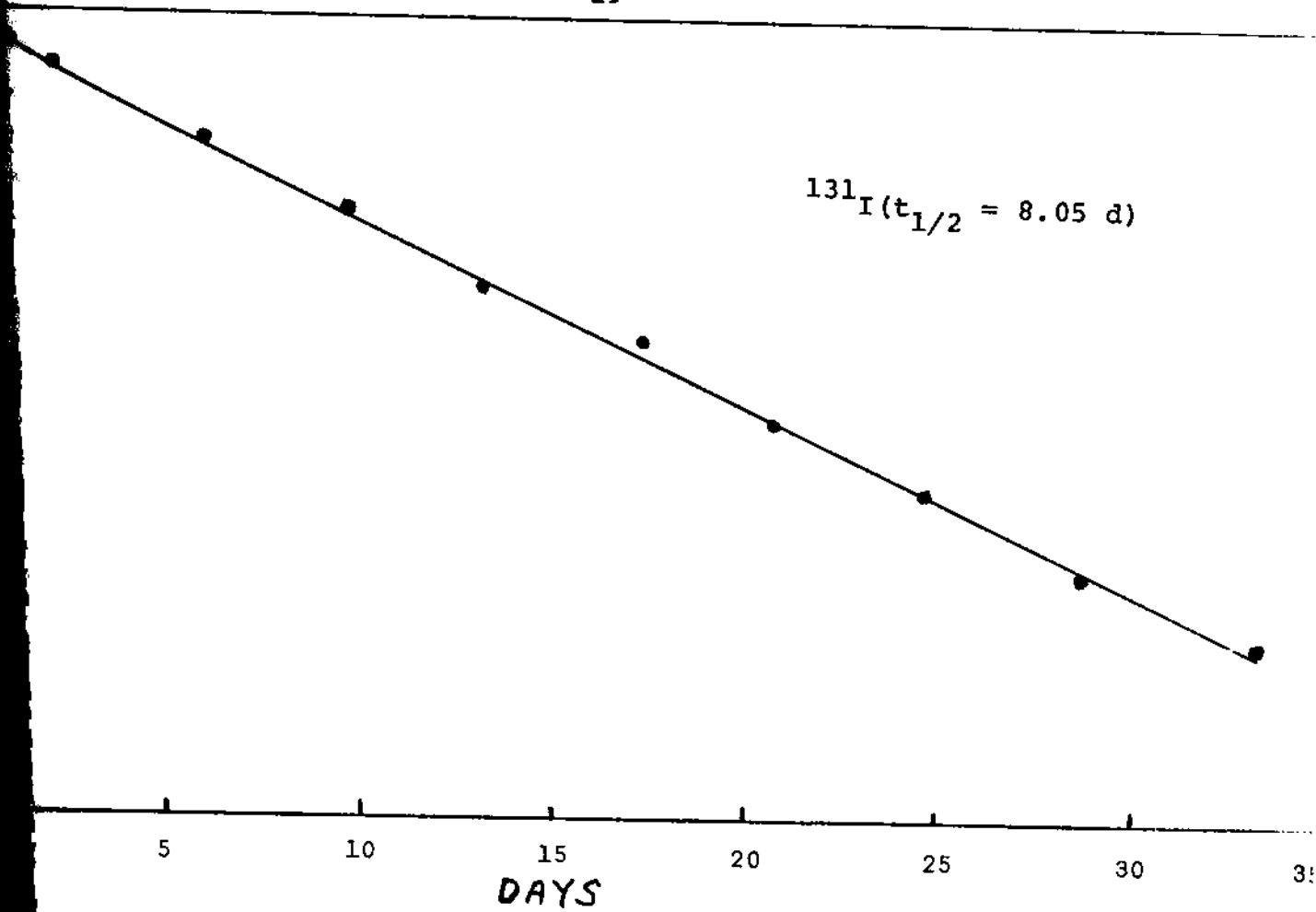


FIG. 5. Decay curve for ^{131}I

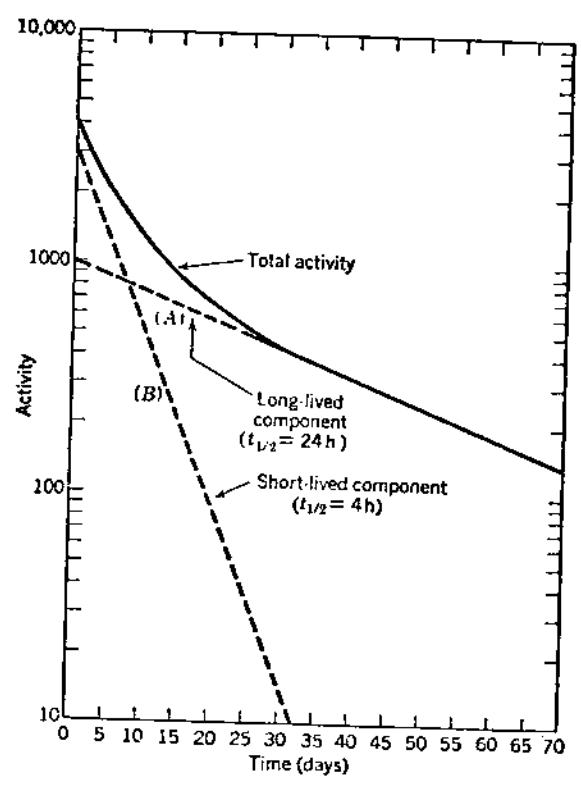


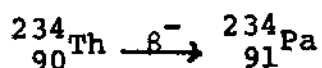
FIG. 6
Analysis of a Decay Curve with Two Components

SECTION V

Parent-Daughter Relationships



Parent Daughter



Parent Daughter

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \quad (18)$$

Where N_1 and N_2 are the numbers of parent and daughter atoms, respectively, at time t .

$$N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1^{\circ} (e^{-\lambda_1 t} - e^{-\lambda_2 t}) + N_2^{\circ} e^{-\lambda t} \quad (19a)$$

At time $t = 0$ $N_2^{\circ} = 0$

$$N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1^{\circ} (e^{-\lambda_1 t} - e^{-\lambda_2 t}) \quad (19b)$$

A. Case I $t_{1/2}$ (parent) < $t_{1/2}$ (daughter)

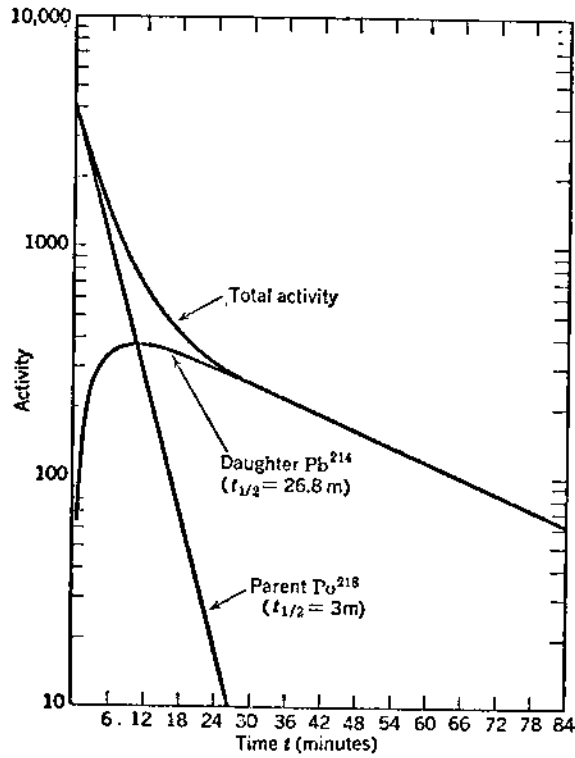


FIG. 7 Growth and decay curves for $^{218}\text{Po} \rightarrow ^{214}\text{Pb}$ system

B. Case II Transient Equilibrium: $t_{1/2}$ (parent) $>$ $t_{1/2}$ (daughter)

$$N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1 \quad (20)$$

$t_{1/2}(p)/t_{1/2}(d)$ roughly between 1 and 100

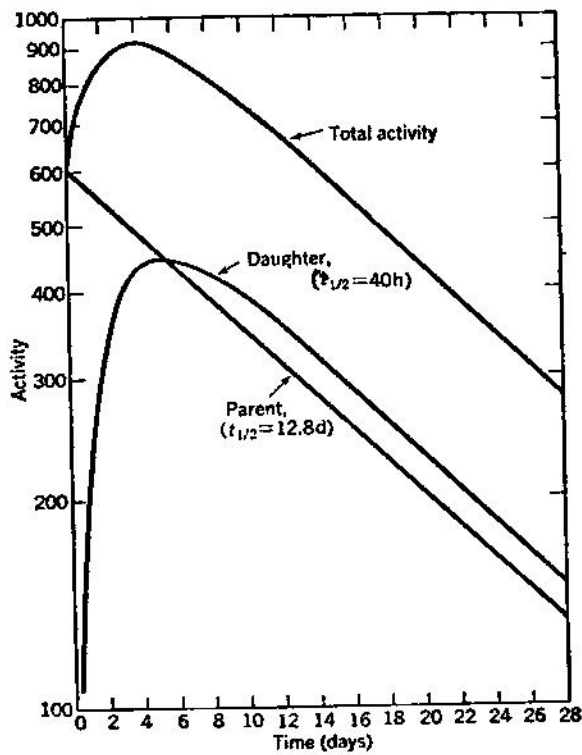


FIG. 8 Transient equilibrium for ¹⁴⁰Ba-La system.

*Always: Growth of daughter with half life of shorter of the two
Decay of daughter with the halflife of the longer*

C. Case III Secular Equilibrium: $t_{1/2}(\text{parent}) \gg t_{1/2}(\text{daughter})$

$$N_2 = \frac{\lambda_1}{\lambda_2} N_1 \quad ; \quad \underline{A_2 = A_1}$$

$$t_{1/2}(\text{p})/t_{1/2}(\text{d}) > 100 \quad (21)$$

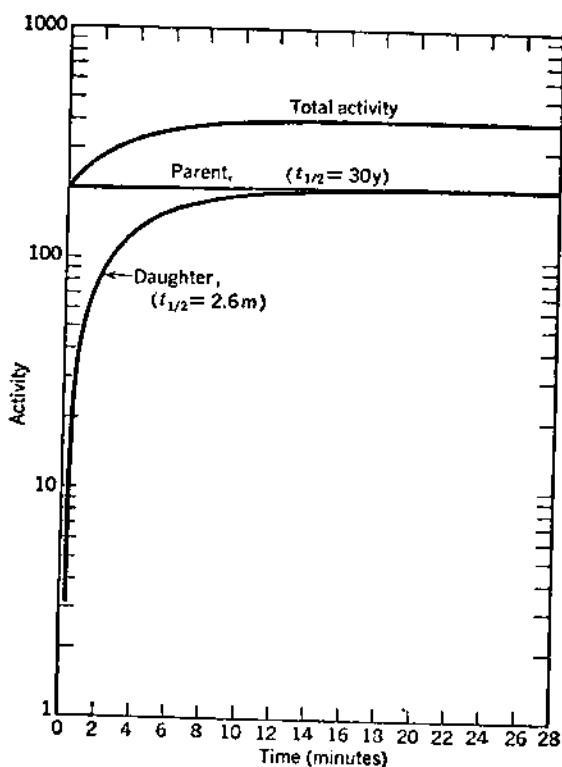


FIG. 9 Secular equilibrium for ^{137}Cs - ^{137}Ba system.

SECTION VI

The Chart of Nuclides

Much data on the radioactivity of nuclides is summarized in the Chart of Nuclides. You should familiarize yourself with how to find nuclides on the chart and extract the desired information.

SYMBOLS

Chemical Element

H
1.0080
~33

- Symbol
- Atomic Weight (Chemical Scale)
- Thermal Neutron Absorption Cross Section in Barns

Stable

Pd 106
26.7
#1.074118
107.8378

- Symbol, mass number
- Percent Abundance
- Activation Cross Sections in Barns to two isomers
- Mass (Physical Scale)
- Fission Product, Slow Neutron Fission of U²³⁵

Artificially Radioactive

Ge 73
5h
β ⁻ 14
(γ, 0.54, 0.130)
E 1.5

- Symbol, Mass Number
- Half-Life
- Mode of Decay, Radiation, and Energy in Mev.
- () Indicate Radiations from Short-Lived Daughter
- Disintegration Energy in Mev.

Naturally occurring or Otherwise Available but Radioactive

Pt 190
0.012
~10 ¹⁰ y 171
~3.3 (P)
~90

- Symbol, Mass Number
- Percent Abundance
- Half-Life. ? Indicates Radioactivity Uncertain
- Mode of Decay, Radiation and Energy in Mev.
- Thermal Neutron Absorption Cross Section in Barns

Member of Naturally Radioactive Decay Chain

Em 220
Tn 52s
6.28, 5.75
220.000

- Half-Life
- Mode of Decay, Radiations and Energies in Mev.
- Mass (Physical Scale)

Two Isomeric States One Stable

Sn 117
14 d 7.6
17.059
γ 162
~118.9405

Radioactive upper isomer Stable Lower Isomer

- Symbol, Mass Number
- Percent Abundance
- Mass (Physical Scale)

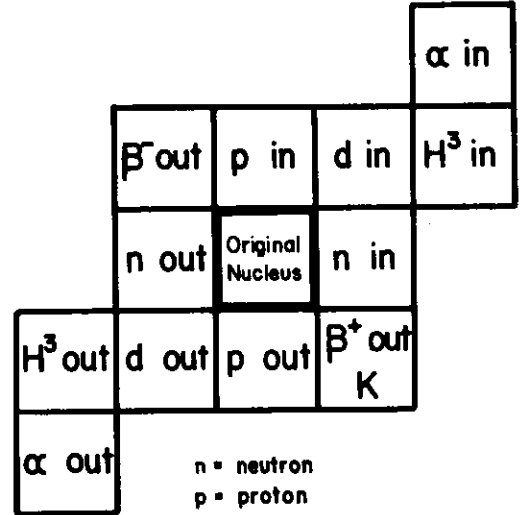
Two Isomeric States Both Radioactive

Hg 193
12 h 5h
17.101 K
γ 0.59
γ 0.38
K
0.32-1.63

Radioactive Upper Isomer Radioactive Lower Isomer

- Symbol, Mass Number
- Half-Lives
- Modes of Decay, Radiations and Energies in Mev.

Relative Locations of the Products of Various Nuclear Processes



- n = neutron
- p = proton
- d = deuteron
- α = alpha particle
- β⁻ = negative beta particle
- β⁺ = positive " "
- K = K-electron capture

RADIATIONS AND DECAY

- α alpha particle
- β⁻ negative beta particle
- β⁺ positive beta particle
- γ gamma ray
- SF spontaneous fission
- n neutron
- e⁻ internal conversion electron
- K electron capture
- IT isomeric transition
- D radiation delayed
- E disintegration energy

TIME

- μs microsecond
- s second
- m minute
- h hour
- d day
- y year

Displacements caused by Nuclear Bombardment Reactions

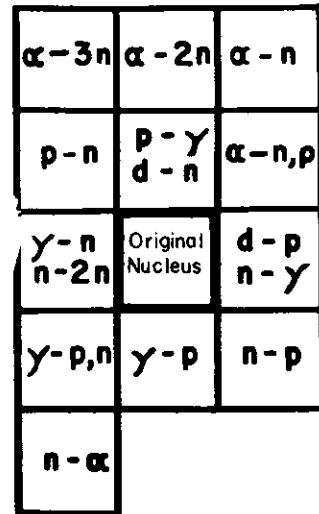


FIGURE 10

Chart of the Nuclides as prepared by General Electric Co. - symbols

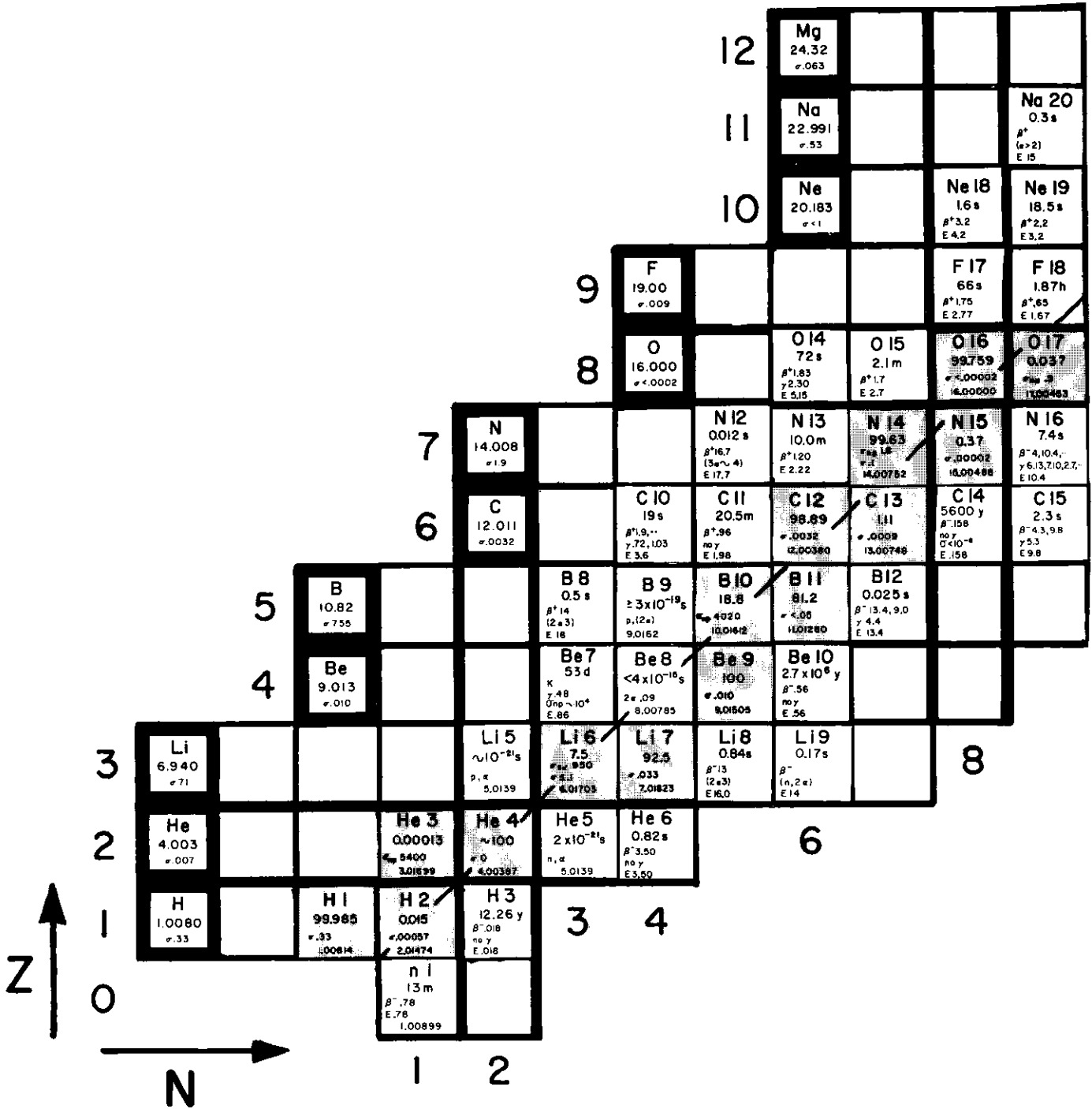


FIGURE 11
 Chart of the Nuclides: Z=1→12; N=0→9

SECTION VII

Problems

1. Calculate the decay energy for the possible decay paths and predict the most probable mode of decay.

	amu
a.	
$^{22}_{11}\text{Na}$ → $^{22}_{12}\text{Mg}$	$^{22}_{12}\text{Mg} = 21.9998$
$^{22}_{11}\text{Na}$ → $^{22}_{10}\text{Ne}$	$^{22}_{11}\text{Na} = 21.9944$
	$^{22}_{10}\text{Ne} = 21.9914$
b.	
$^{32}_{15}\text{P}$ → $^{32}_{16}\text{S}$	$^{32}_{16}\text{S} = 31.9721$
$^{32}_{15}\text{P}$ → $^{32}_{14}\text{Si}$	$^{32}_{15}\text{P} = 31.9739$
	$^{32}_{14}\text{Si} = 31.9740$
c.	
$^{141}_{58}\text{Ce}$ → $^{141}_{59}\text{Pr}$	$^{141}_{59}\text{Pr} = 140.9076$
$^{141}_{58}\text{Ce}$ → $^{141}_{57}\text{La}$	$^{141}_{58}\text{Ce} = 140.9082$
	$^{141}_{57}\text{La} = 140.9108$
d.	
$^{239}_{93}\text{Np}$ → $^{239}_{92}\text{U}$	$^{239}_{93}\text{Np} = 239.0529$
$^{239}_{93}\text{Np}$ → $^{239}_{94}\text{Pu}$	$^{239}_{92}\text{U} = 239.0543$
$^{239}_{93}\text{Np}$ → $^{235}_{91}\text{Pa}$	$^{239}_{94}\text{Pu} = 239.0521$
	$^{235}_{91}\text{Pa} = 235.0454$
e.	
$^{249}_{97}\text{Bk}$ → $^{249}_{96}\text{Cm}$	$^{249}_{96}\text{Cm} = 249.0758$
$^{249}_{97}\text{Bk}$ → $^{249}_{98}\text{Cf}$	$^{249}_{97}\text{Bk} = 249.0749$
$^{249}_{97}\text{Bk}$ → $^{245}_{95}\text{Am}$	$^{249}_{98}\text{Cf} = 249.0763$
	$^{245}_{95}\text{Am} = 245.0663$

2. ${}_{39}^{90}\text{Y}$ decays by β^- emission to ${}_{40}^{90}\text{Zr}$. If the decay energy, Q , is measured to be -2.79 MeV and the mass of ${}_{40}^{90}\text{Zr}$ is determined by mass spectrometry to be 89.9047 , what is the mass of ${}_{39}^{90}\text{Y}$?
3. In natural indium, ${}_{49}^{115}\text{In}$ is found in 95.8% abundance; in natural tin, ${}_{50}^{115}\text{Sn}$ is found in 0.38% abundance. Which would be expected to be unstable with respect to the other, and what approximate lower limit can be given the half life, since this nuclide is still found in nature, if the age of the elements is assumed to be 7×10^9 years?

$$\text{Masses } \quad {}_{49}^{115}\text{In} = 114.9039 \text{ amu} \quad {}_{50}^{115}\text{Sn} = 114.9033 \text{ amu}$$

4. What is the mass of ${}_{92}^{235}\text{U}$ if the decay energy for the alpha decay of ${}_{94}^{239}\text{Pu}$ is 5.21 MeV?

$${}_{2}^{4}\text{He} = 4.0026 \text{ amu}$$

$${}_{94}^{239}\text{Pu} = 239.0521 \text{ amu}$$

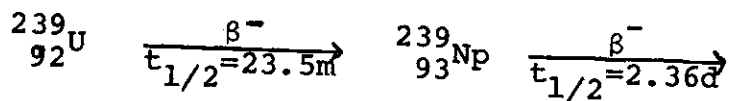
5. Calculate the number of grams and the number of radioactive atoms in a millicurie of:

a. ${}_{17}^{36}\text{Cl} \quad t_{1/2} = 3 \times 10^5 \text{ y}$

b. ${}_{33}^{76}\text{As} \quad t_{1/2} = 27.6 \text{ h}$

c. ${}_{24}^{51}\text{Cr} \quad t_{1/2} = 27.8 \text{ d}$

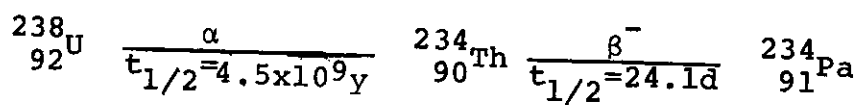
6. What is the count rate of a sample nine half lives after a rate of 10^7 cpm is measured?
7. If a freshly prepared sample of $^{239}_{92}\text{U}$ of 10^6 dpm is allowed to stand for 6 hours, what will the total disintegration rate be at the end of that time?



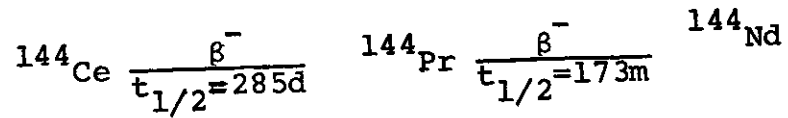
8. The following data were taken in an appropriate counter. Resolve these data into two activities and ascertain their half lives.

<u>t (hr)</u>	<u>dpm</u>	<u>t (hr)</u>	<u>dpm</u>	<u>t (hr)</u>	<u>dpm</u>
0	7000	0.10	2300	30	590
1	6000	12.5	1850	35	450
2	5300	15	1500	40	340
3	4600	17.5	1250	45	260
5	3800	20	1060	50	200
7.5	2860	25	780		

9. If a kilogram of $^{238}_{92}\text{U}$ is purified and then set aside for 2 months, what weight of $^{234}_{90}\text{Th}$ will be present in the uranium sample at the end of that time?

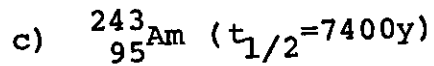
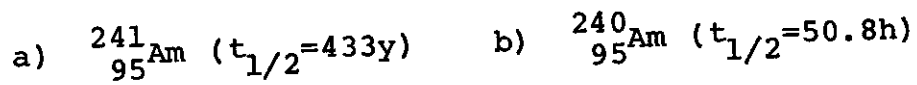


10. For the decay chain



when the ${}^{144}\text{Ce}$ and ${}^{144}\text{Pr}$ are in secular equilibrium, what is the weight of ${}^{144}\text{Pr}$ per gram ${}^{144}\text{Ce}$?

11. Calculate the specific activity per gram of the isotopes for the following



How does the specific activity vary with half-life?