PART VI

Counting Statistics and Calibration

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SECTION I

Counting Statistics

A. Definition of Errors

Determinate Error - usually systematic and constant. Can usually be eliminated by redesign of experiment.

Random error - neither systematic or constant. Must be evaluated by statistical methods.

Table 1 Notation

$\bar{n}$ = arithmetic mean of all the measured values

n = a measured value of total counts

P(n) = probability of occurrence of the value n

σ = standard deviation

p = probable error

R = count rate in counts per minute

t = time of measurement

N = number of measurements on a given sample

Subscripts

s = sample only

b = background

t = total (sample + background)

B. Distribution of Random Events

Binomial Distribution Law - correctly expresses probability but very difficult to use. Common practice is to use:

1. Poisson Distribution

$$P(n) = \frac{\bar{n}^{+n} e^{-\bar{n}}}{n!} \qquad (1)$$

A Poisson Distribution is not symmetric about $\bar{n}$, the average value, for low values of $\bar{n}$. See Fig. 1.

2. Gaussian Distribution

$$P(n) = \frac{1}{(2\pi n)^{\frac{1}{2}}} \; e^{\frac{-(\bar{n}-n)^2}{2\bar{n}}} \qquad (2)$$

A Gaussian Distribution is symmetric about $\bar{n}$.

Note: If $\overline{n} \gtrsim 100$ the Gaussian function may be substituted for the Poisson Distribution with no appreciable error.

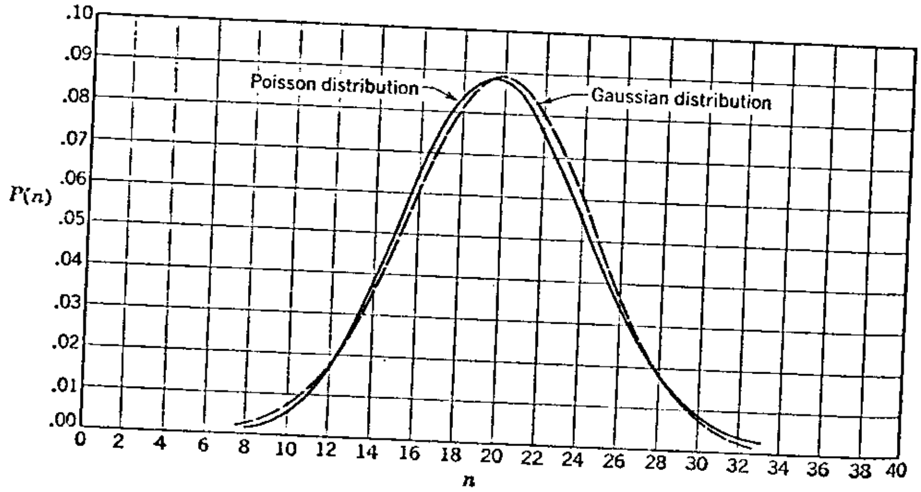

FIG. 1. The Poisson and Gaussian distributions for $\overline{n} = 20$.

a. Standard Deviation (σ) for total count using Gaussian Distribution

$$\sigma = \left[\frac{1}{N} \sum_{i=1}^{N} (\overline{n}-n_i)^2\right]^{\frac{1}{2}} = n^{\frac{1}{2}} \qquad (3)$$

Note: σ corresponds to a 68% probability

Total count = rate x total time

Ex: Gross sample count rate = 1000 cpm

total time = 10 m

Total Count = 1000 cpm x 10 m = 10,000 counts

$\sigma = (n)^{1/2} = (10{,}000)^{1/2} = \pm 100$

Therefore: Counts $\pm \sigma$ = 10,000 $\pm$ 100 counts

or 9900 - 10,100 counts range for 68% probability.

Consequently, if this sample is counted 100 times, 68 times the total count will be between the limits of 9900-10,100 counts and 32 times, outside of these limits.

b. Standard Deviation (σ_R) in the counting rate (R)

$$\sigma_R = \frac{(n)^{1/2}}{t} = \left(\frac{R}{t}\right)^{1/2} \qquad (4)$$

σ_R for previous example:

$$\frac{(10{,}000)^{1/2}}{10} = \frac{1000}{10} = \pm 10 \text{ cpm}$$

c. Confidence Levels and K

Confidence level - expresses the % probability of the true value occuring within certain limits

$$K = \text{number of standard deviations} = \frac{|\bar{n}-n|}{\sigma}$$

Table II. Table of Constants of Relative Error

Confidence Level (%)=(b)	K	
50	0.6745	Probable Error
68.27	1.0000	Standard deviation (1σ)
90.00	1.6449	Nine tenths Error
95.00	1.9600	Ninety-five hundredths error
95.45	2.0000	2σ
99.00	2.5758	Ninety-nine hundredths error
99.73	3.0000	3σ
99.9937	4.0000	4σ

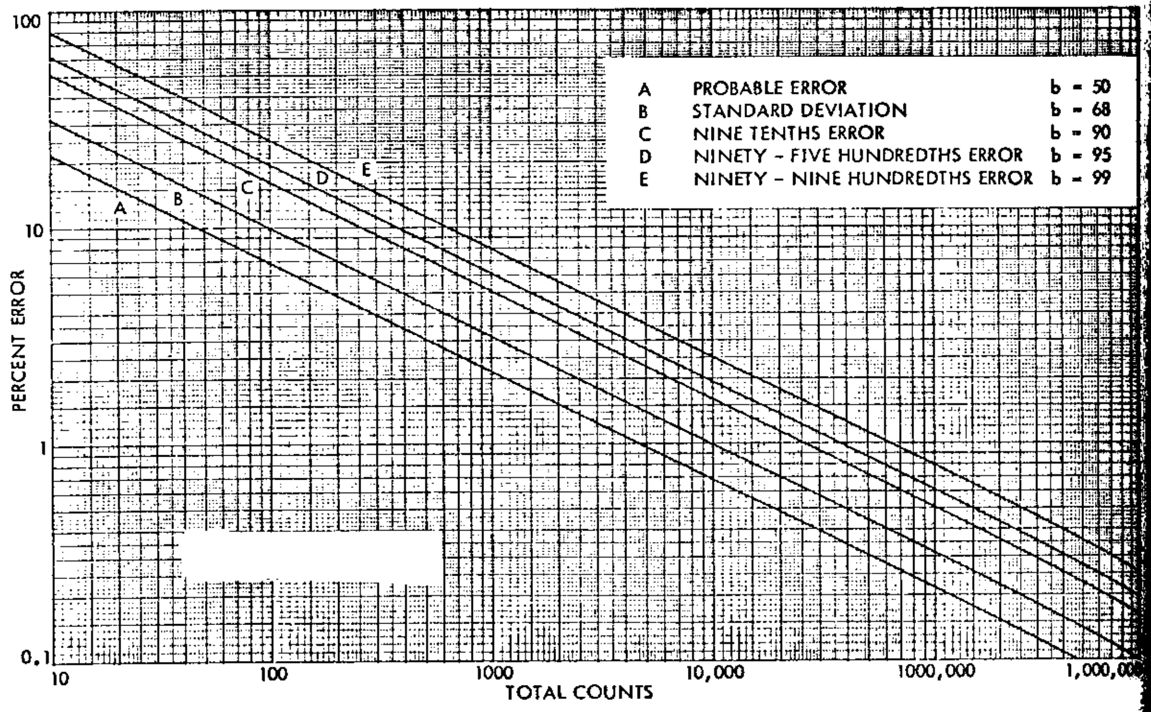

FIG. 2. Percent Error versus Gross Count

To determine the limits, Δn, corresponding to a certain confidence level for a total count of n

$$\Delta n = K_i \sigma = K_i (n)^{1/2} \qquad (5)$$

where K_i is the K corresponding to the particular confidence level

Ex: total count n = 10,000

For probable error (or 50% confidence level)

$\Delta n = 0.675\ (n)^{1/2} = 0.6745\ (10{,}000)^{1/2} = \pm\ 67$

For 95% confidence level

$\Delta n = 1.96\ (n)^{1/2} = (1.96)(10{,}000)^{1/2} = \pm\ 196$

C. Error in Total Counting System

Total count = sample count + background count

∴ Sample count = total count - background count

1. (a) Standard deviation for total sample alone.

$$\sigma_s = (\sigma_t^2 + \sigma_b^2)^{1/2} = (n_t^2 + n_b^2)^{1/2} \qquad (6a)$$

Frequently $\sigma_b << \sigma_t$

$$\therefore \sigma_s = (\sigma_t{}^2)^{1/2} = \sigma_t \qquad (6b)$$

(b) Standard deviation for sample counting rate alone.

$$\sigma_{R_s} = (\sigma_{R_t}^2 + \sigma_{R_b}^2)^{1/2} = \left(\frac{R_t}{t_t} + \frac{R_b}{t_b} \right)^{1/2} \qquad (7a)$$

Frequently $\sigma_{R_b} << \sigma_{R_t}$

$$\therefore \sigma_{R_s} = \left(\sigma^2_{R_t}\right)^{1/2} = \sigma_{R_t} \qquad (7b)$$

3. Optimization of counting conditions.

$$\frac{t_b}{t_t} = \left(\frac{R_b}{R_t}\right)^{1/2} \qquad (8)$$

Ex: Suppose you are using a counter with a background count rate of 50 cpm and are only allotted one hour to count your sample which has a gross count rate of 2000 cpm. How should you proportion your allotted time between sample and background for minimum error?

$R_b = 50$ cpm $\qquad R_t = 2000$ cpm $\qquad t_t = 60$ min

$$\frac{t_b}{t_t} = \left(\frac{R_b}{R_t}\right)^{1/2} = \left(\frac{50 \text{ cpm}}{2000 \text{ cpm}}\right)^{1/2}$$

$$\frac{t_b}{60 \text{ min}} = .158$$

$t_b = 9.48$ min.

∴ You must take a 9.5 min background count and use the remaining time for sample counting (50.5 min).

4. Error in dead time corrections.

For GM counters with count rates greater than 8000-10,000 cpm or proportional counters with count rates greater than 10^5 cpm

$$\sigma_m = (n)^{1/2}\left(\frac{m}{n}\right) \qquad m = \text{true count rate} \qquad (9)$$

5. Calculations for minimum count rates.

$$R_{s(min)} = \frac{1+2F_\sigma(tR_b)^{1/2}}{F_\sigma^2 t} \qquad (10)$$

where $F_\sigma = \frac{\sigma_{R_s}}{R_s}$ and t is total time

Ex: Suppose you are using a counter (50 cpm background) to count your sample (R_t = 200 cpm) and given only one hour to complete the counting. For a desired fractional standard deviation of 1% what is the minimum sample count rate necessary?

$F_\sigma = 0.1 \quad R_b = 50$ cpm $\quad t = 60$ min

$$R_{s(min)} = \frac{1+2(.01)(60 \text{ min X } 50 \text{ cpm})^{1/2}}{(0.1)^2(60 \text{ min})}$$

$$R_{s(min)} = 349 \text{ cpm}$$

Is your experiment valid?

No, you do not have enough sample to count for your allotted time to give you 1% error. You need a sample which has a minimum counting rate of 349 cpm and your sample is only 150 cpm.

$$(R_s = 200 \text{ cpm} - 50 \text{ cpm})$$

6. Calculation for preset time (pst) or preset count (p
for a predetermined accuracy.

$$pst = \frac{K^2 x 10^4}{F^2} \times \frac{1}{R_s} \left[1 + \frac{2R_b}{R_s}\right] \qquad (11)$$

$$psc = R_t \times \frac{K^2 x 10^4}{F^2} \times \frac{1}{R_s} \left[1 + \frac{2R_b}{R_s}\right] \qquad (12)$$

F = desired error as percentage

Ex: Suppose you had a total count rate of 600 cpm and the background rate was 50 cpm.

How long would you have to count your sample to obtain a 2% accuracy at 99% confidence level?

$R_s + R_t - R_b = 600$ cpm $- 50$ cpm $= 550$ cpm

K (for 99% confidence level from Table II) = 2.5758; F = 2

$$pst = \frac{(2.5758)^2 \times 10^4}{(2)^2} \times \frac{1}{550} \left[\frac{1 + 2(50)}{550}\right]$$

pst = 35.6 m

Suppose you counted the same sample to a preset count of 36,000, what would your percent accuracy be at a 95% confidence level?

$$psc = R_t \times \frac{K^2 x10^4}{F^2} \times \frac{1}{R_s} \left[1 + \frac{2R_b}{R_s}\right]$$

$$F = \left[R_t \times \frac{K^2 x10^4}{psc} \times \frac{1}{R_s} \{1 + \frac{2R_b}{R_s}\}\right]^{1/2} \qquad (13)$$

K = 1.96 for 95% confidence level

$$F = \left[600\ \text{cpm} \times \frac{(1.96)^2}{36{,}000} \times 10^4 \times \frac{1}{550} \{1 + \frac{2(50)}{550}\}\right]^{1/2}$$

$$F = 1.17\%$$

Graphical Aids to Counting Statistics

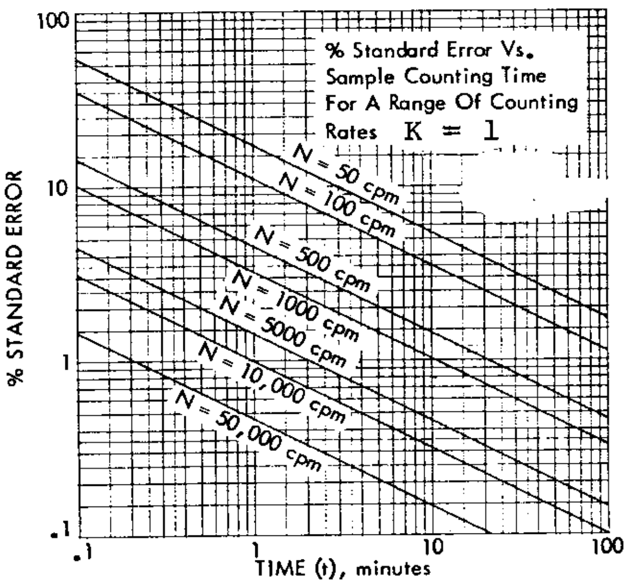

FIG. 3. Percent Error versus Counting Time (R_b=12cpm)

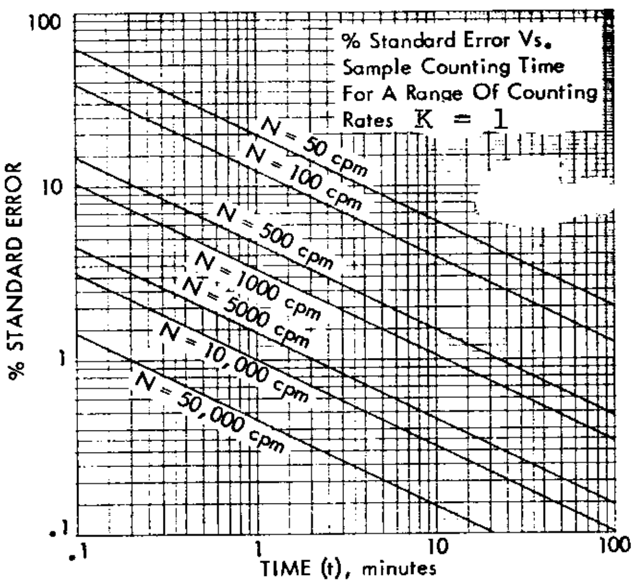

FIG. 4. Percent Error versus Counting Time (R_b=25cpm)

For N = 50 cpm what is the % standard error in a 10 min count for both background counting rates?

Fig. 3 - 5.5%   Fig. 4 - 6.3%

For N = 10,000 cpm and t = 10 min

Fig. 3 - 3.3%   Fig. 4 - 3.3%

7. Relationship of net sample count rate and background count rate.

Use following relationships: R_s = sample counting rate

$$\%F = K \times \frac{100}{(aR_s)^{1/2}} \times C \qquad (14)$$

a = number of samples (or number of replicate counts on same sample)

K = number of standard deviations

F = desired error as %

Where C depends on counting conditions

$$C_1 = \frac{d}{d-1} \quad \text{(Background is counted for a long time to make error in background negligible)} \qquad (15)$$

$$C_2 = \frac{\sqrt{d^2+1}}{d-1} \quad \text{(Preset COUNT is selected)} \qquad (16)$$

$$C_3 = \frac{\sqrt{d^2+d}}{d-1} \quad \text{(Preset time is selected)} \qquad (17)$$

and $d = \frac{R_t}{R_b}$

TABLE III

d	C_1	C_2	C_3
1.1	11.000	14.866	15.20
1.2	6.000	7.813	8.124
1.3	4.333	5.477	5.763
1.4	3.500	4.301	4.650
1.5	3.000	3.605	4.025
1.8	2.250	2.574	2.861
2	2.000	2.236	2.449
3	1.500	1.581	1.732
4	1.333	1.378	1.491
5	1.250	1.275	1.369
6	1.200	1.216	1.296
7	1.167	1.178	1.245
8	1.143	1.152	1.212
9	1.125	1.132	1.173
10	1.111	1.116	1.164
25	1.0417	1.0425	1.063
50	1.0200	1.0206	1.031
75	1.0135	1.0136	1.020
100	1.0101	1.0101	1.015
250	1.004	1.004	1.006
500	1.002	1.002	1.003
750	1.001	1.001	1.002
1000	1.001	1.001	1.0015

Ex: When preset count is selected what is the percent error which corresponds to the following conditions:

$R_t = 1250$ cpm $R_b = 50$ cpm $a = 1$

K = one sigma or 1.000 (from Table II)

$$R_s = R_t - R_b = 1200 \text{ cpm}$$

$$\%F = K \times \frac{100}{(aR_s)} 1/2 \times C_2$$

$$d = \frac{R_t}{R_b} = \frac{1250}{50} = 25$$

From Table III, d = 25 corresponds to C_2 = 1.0425

$$\%F=(1.000) \times \frac{100}{(1\times1200)^{1/2}} \times 1.0425$$

$$\%F = 3.01$$

8. Choice of counting technique

Advantage of greater selectivity is accompanied by disadvantage of higher background

To determine which system will require shorter counting time for the same error

$$\frac{t}{t'} = \frac{\left[\frac{(aR_t)^{1/2} + (R_b)^{1/2}}{R_s}\right]^2}{\left[\frac{(aR'_t)^{1/2} + (R'_b)^{1/2}}{R'_s}\right]^2} \qquad (18)$$

a = number of samples

$R_s = R_t - R_b$ (in cpm)

t = time (in min)

Unprimed - first system

Primed - second system

Ex: In a Geiger-Muller Counter, a sample registered a total count of 100 cpm with a background of 30 cpm. In a proportional counter the same sample gave 125 cpm with a 50 cpm background. If 3 such samples are to be counted which counter will give the same error in less time?

GM $R_t = 100$ cpm

$R_b = 30$ cpm

$R_s = 70$ cpm

Proportional $R_t' = 125$ cpm

$R_b' = 50$ cpm

$R_s' = 75$ cpm

a = 3

$$\frac{t}{t'} = \frac{\left[\dfrac{(3\text{x}100)^{1/2}+(30)^{1/2}}{70}\right]^2}{\left[\dfrac{(3\text{x}125)^{1/2}+(50)^{1/2}}{75}\right]^2} = \frac{.106}{.124} = .853$$

The ratio of $\frac{t_{\tau}}{t}$ is less than one, (i.e. t'>t) and therefore the GM counter will provide the same error as the proportional counter in less time.

9. Errors due to electronic or mechanical misbehavior and data rejection

Two tests

a. Repetitive counts of sample

$$K = \frac{n-n_i}{\sigma_{\bar{n}}} \qquad (19)$$

If K>3.5 - error non-statistical

Ex: Suppose the same sample was counted five times with the following results:

1. 504
2. 500
3. 502
4. 575
5. 502

Can any of your data be rejected?

The fourth count appears to be much too high; let's check:

504	$\bar{n} = \frac{2008}{4} = 502 \quad \sigma_{\bar{n}} = (\bar{n})^{1/2} = 22.4$
500	
502	$n_1 = 575$
502	
2008	$K = \frac{502-575}{22.4} = 3.26$

K<3.5 so the fourth count may not be rejected by this test.

b. Chi-Square Test

$$\chi^2 = \sum_{i=1}^{N} \frac{[(\text{observed value})_i - (\text{expected value})_i]^2}{(\text{expected value})_i}$$

If the average value is small (e.g., less than 100) you must use the Poisson Distribution to calculate the expected value.

Typically $\bar{n}>100$ so the Gaussian Distribution can be used. In this case the expected value is identical to $\bar{n}$, the average value.

Thus for $\bar{n}\geq 100$

$$\chi^2 = \frac{1}{\bar{n}} \sum_{i=1}^{N} (n_i-\bar{n})^2 \qquad (21)$$

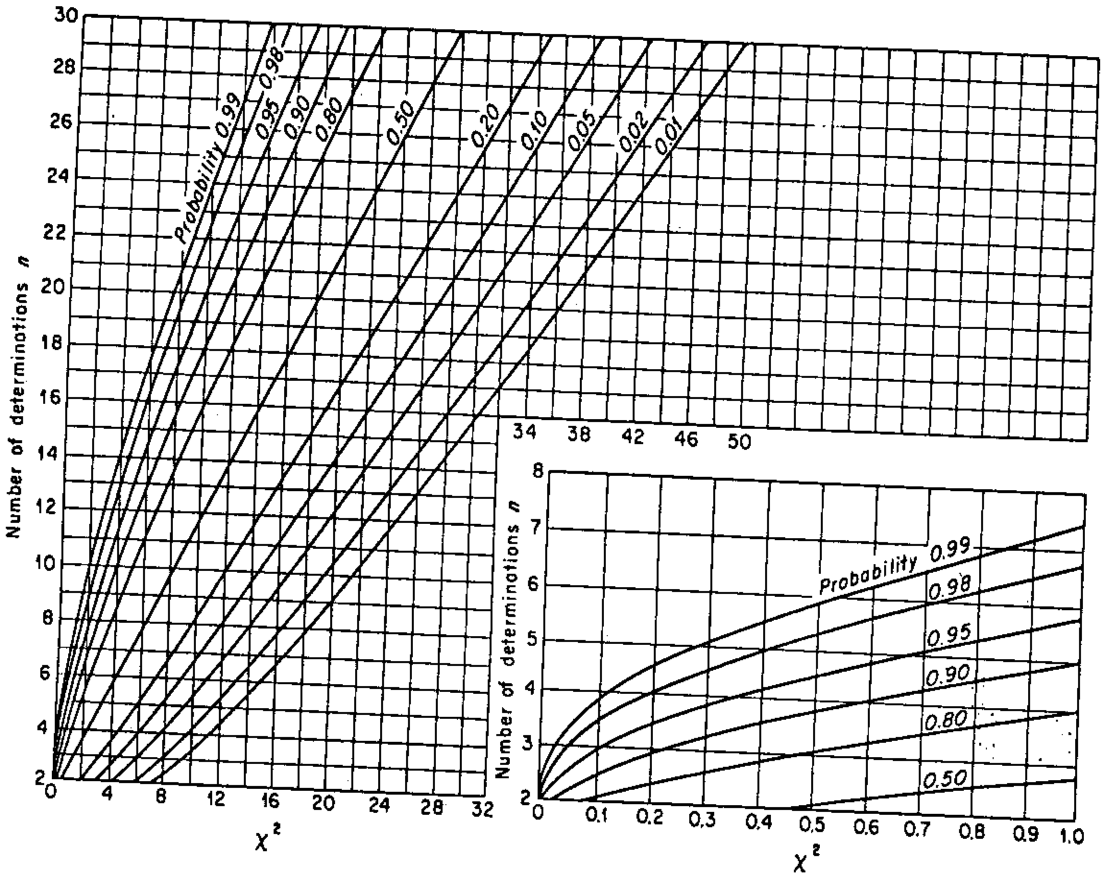

FIG. 5. The χ^2 distribution. The probability that the variations in a series of counting determinations are due to the randomness of the disintegration process. n is the number of determinations.

Example: A G-M counter was tested to see if it was operating properly by taking background counts in eight equal time intervals. The results are listed below

Interval	Count = n
1	312
2	298
3	318
4	309
5	330
6	306
7	291
8	326

The total number of counts observed = 2490 counts

$$\bar{n} = \frac{2490}{8} = 311$$

$\overline{n} > 100$ so eqn (20) was used

Interval	n	$n-\overline{n}$	$(n-\overline{n})^2$
1	312	1	1
2	298	-13	169
3	318	7	49
4	309	-2	4
5	330	19	361
6	306	-5	25
7	291	-20	400
8	326	15	225
TOTAL 8	2490	0	1234

$$\chi^2 = \frac{1234}{311} = 3.97$$

Using Fig. 5 with N=8 and χ^2=1.29 the probability ∿ 0.80

Therefore, these 8 counts indicate that the probability that G.M. counter is operating properly is 80%.

Generally felt that better criterion is to reject values deviating from mean by more than 2σ (4.5% probability) or 3σ (0.27%).

SECTION II

Radioactive Standards and Calibration

A. Determination of Absolute Counting Rate - see Part V - Section II.

B. Standards and Calibration Services

More information about Standard Reference Materials can be obtained from:

"Standard Reference Materials", NBS, Spec. Publ. 260, Gov't. Printing Office, Washington, D.C. (semiannuall

or "Calibration and Test Services of the National Bureau of Standards", NBS, Spec. Publ. 250, Gov't. Printing Office, Washington, D. C. (1970) (revised pages and new additions as needed.)

Guidelines for the user's of Radioactivity Standards can be found in:

"User's Guide for Radioactive Standards": NAS-NS 3115(1974).

SECTION III

Problems

1. A sample with a gross counting rate of 5,000 cpm is counted for half an hour. What is the interval about the total count which corresponds to a 68% probability?

2. A sample counted for 15 minutes gave 9,000 total counts. A 30 minute background count registered 1200 counts. Calculate the count rate of the sample alone with its standard deviation and its probable error.

3. If only 30 minutes of counting time are available, calculate the desirable division of time between sample and background for minimum error if the background is approximately 30 cpm and total rate 100 cpm. What will the standard deviation for the sample be, expressed as a percentage error?

4. For a series of 100 counts, $\bar{n}$ is 100 cpm. How many will give 127 cpm, 83 cpm, 100 cpm?

5. In a Geiger-Muller counter, a sample registered a total of 60 cpm with a background of 30 cpm. In a proportional counter the same sample gave 95 cpm with a background

of 60 cpm. If 3 such samples are to be counted, which system will give the same error in less time?

6. The following counting data were collected per minute with one sample: 3308, 3277, 3411, 3080, 3580, 3425, 3207, 3436, 3328, 3363 counts. Should any of these counts be rejected if it is desired to calculate a good average count rate?

7. A Geiger-Muller counter has a background of 18 cpm with a lead shield. Unshielded, the background rises to 48 cpm. For a sample which counts 60 cpm above background, what are the necessary counting times in the shielded and unshielded arrangements to obtain 5% probable error?

8. The same sample is counted with two different Geiger-Muller counters to determine whether their absolute sensitivity is the same. Counter 1 gave 500 counts in the same time interval that Counter 2 give 525 counts. Is this a statistically significant difference? Would the difference be significant if the two counts were 500 and 590?

9. Calculate the total number of counts that must be collected in each case for a count rate of 500 cpm to give probable errors of 0.1%, 0.5%, 1%, 5%, 10% and 50%.

10. Two successive counts in a proportional counter of the same sample for the same interval gave values of 5440 and 5600. Can the counter be considered to be operating normally? What would be the most probable conclusion if the two counts were 5440 and 5750?