

PART VIII

Induced Nuclear Reactions

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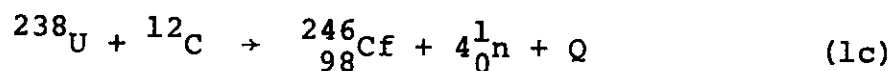
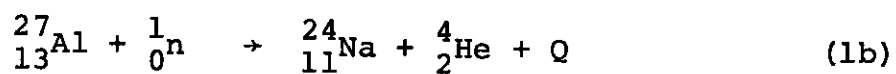
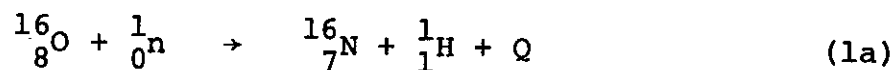
SECTION I

Introduction to Induced Nuclear Reactions

Induced Nuclear Reaction - Process by which changes are induced in nuclei by the interaction with other particles or sufficient energy.

A. Notation for nuclear reactions

1. Formal notation



2. Shorthand notation



A - target nucleus, irradiated material

b - projectile, bombarding particle

c - emitted particles

D - product or residual nucleus



B. Schematic reaction diagram

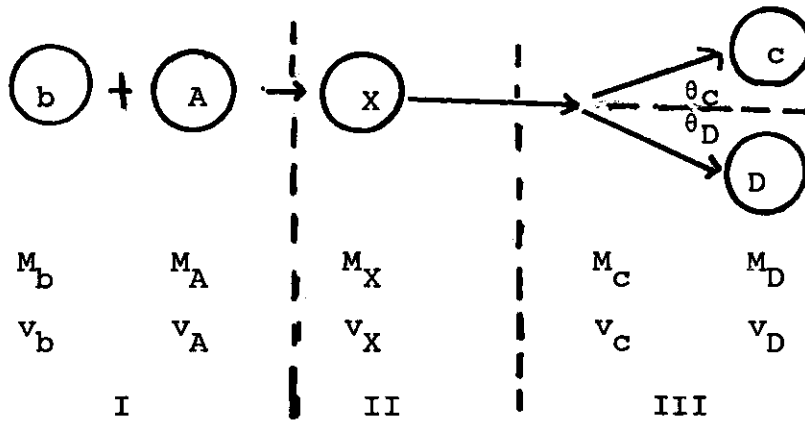


Figure 1.

I = collision stage

II = compound nucleus stage (maybe 10^{-14} s)

III = product formation stage

C. Important quantities conserved

1. Mass number; $A_A + A_b = A_C + A_D$ (3)

2. Nuclear charge; $Z_A + Z_b = Z_C + Z_D$ (4)

3. Total energy; $E_K =$ kinetic energy, $E_M =$ mass energy

$$(E_K + E_M^0)_A + (E_K + E_M^0)_b = (E_K + E_M^0)_C + (E_K + E_M^0)_D \quad (5)$$

4. Momentum: $M_A v_A + M_b v_b = M_X v_X = M_C v_C \cos \theta_C + M_D v_D \cos \theta_D$

and $M_C v_C \sin \theta_C = M_D v_D \sin \theta_D$ (7)

also, $M_b v_b = (M_A + M_b) v_X$ (assume $v_A = 0$)

$$E_K(X) = \left(\frac{M_A}{M_A + M_b} \right) E_{Kb} \quad (8)$$

5. Angular momentum.

D. Types of projectiles

1. Neutrons

Slow neutrons $E \leq 1\text{keV}$

Thermal neutrons; Maxwellian distribution of

velocities which at room temperature has a most probable energy, of 0.025 eV.

Fast neutrons; $E > 0.5$ MeV

Obtain neutrons in nuclear reactors or special "neutron generators" which use reactions such as ${}^2_1\text{H}(d,n){}^3_1\text{H}$.

2. Charged Particles

Protons; ${}^1_1\text{H}$ or p

Deuterons; ${}^2_1\text{H}$ or d

Tritons; ${}^3_1\text{H}$ or t

Alpha particles; ${}^4_2\text{He}$ or α

Heavy ions; any charged particle heavier than α 's, such as ${}^3_3\text{Li}$, ${}^6_6\text{C}$, ${}^{36}_{36}\text{Kr}$, ${}^{54}_{54}\text{Xe}$.

Obtain charged particles from accelerators such as cyclotrons, synchrotrons, linacs, hylacs, etc.

E. Threshold Energy

E_{th} is minimum energy required for a nuclear reaction.

If $Q > 0$, exoergic, no threshold exists

If $Q < 0$, endoergic

If b is projectile, then the denominator should be M_a .

$$E_{th} = -Q \left(\frac{M_b + M_a}{M_b} \right) \quad (9)$$

In bombardment by a charged particle such as p, d, α , or heavy ions, kinetic energy necessary to induce reaction $> E_{th}$ (due to coulomb repulsion)

For neutrons, no coulomb barrier so reaction can be induced by neutrons of zero kinetic energy.

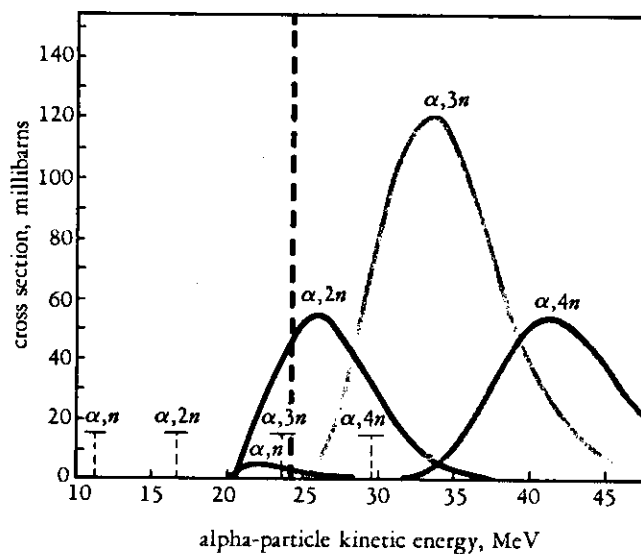


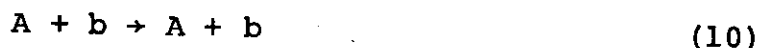
Fig. 2. Formation curves ("excitation functions") for $^{232}\text{Th} + \alpha$. Threshold energies are indicated by short dashed lines while the heavy dashed line indicates the calculated coulomb barrier.

SECTION II

Types of Nuclear Reactions

A. Elastic scattering

No change in identity of particle or target; target remains in ground state; total kinetic energy unchanged.



$$E_{KA} + E_{Kb} = E'_{KA} + E'_{Kb}$$

B. Inelastic scattering

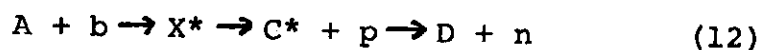
No change in identity of particle or target but target raised to excited state.



$$E_{KA} + E_{Kb} > E'_{KA} + E'_{Kb}$$

C. Spallation

After capture, nucleons emitted; e.g. $A(b, pn)D$:



D. Fission

After capture, nucleus splits into 2 nuclei.

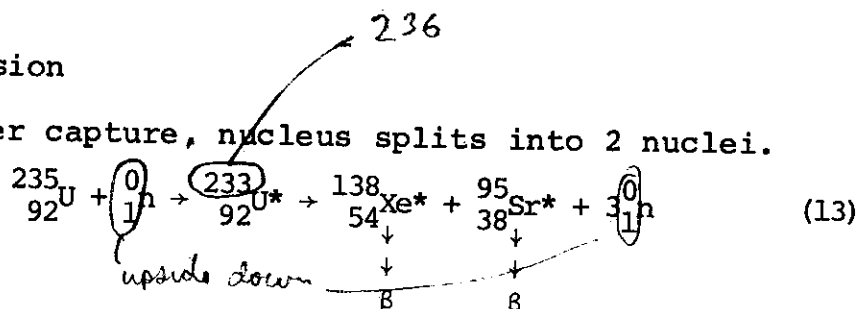


TABLE I. Approximate Fission Energy Balance

	Fission Energy, m.e.v.
Kinetic energy of fission fragments	170
Energy of gamma rays	16
Energy of beta decay	9
Kinetic energy of emitted neutrons	5
Total	200

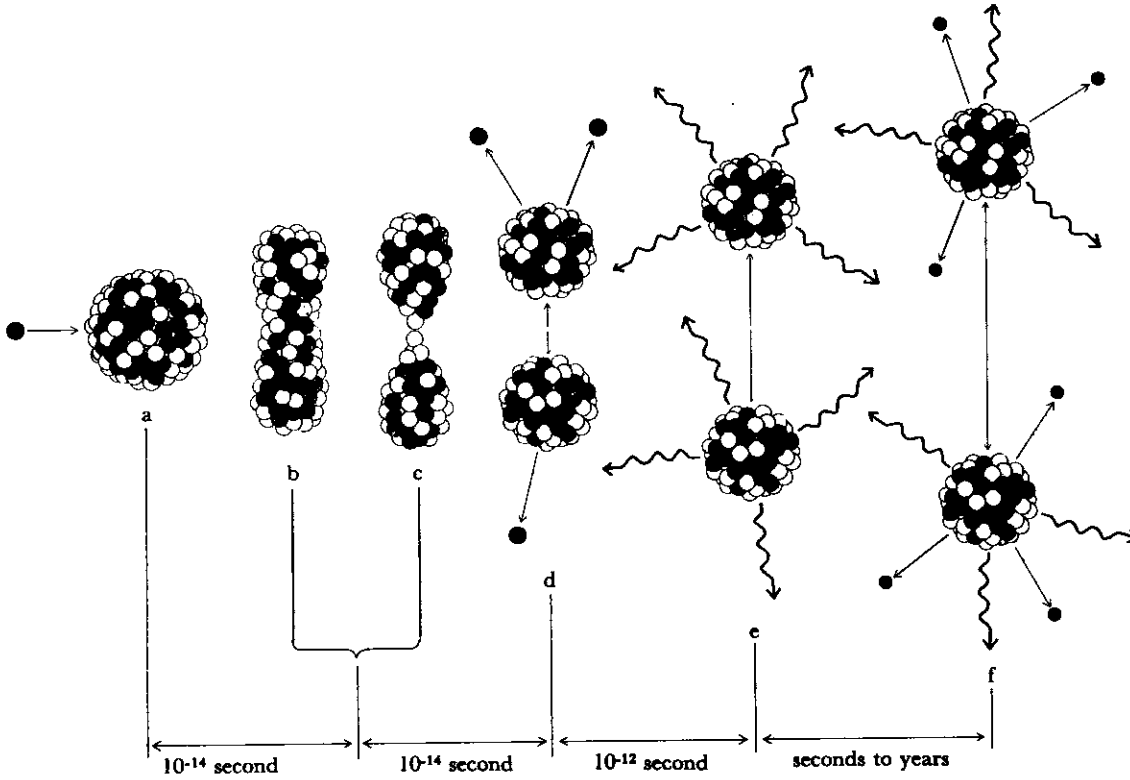
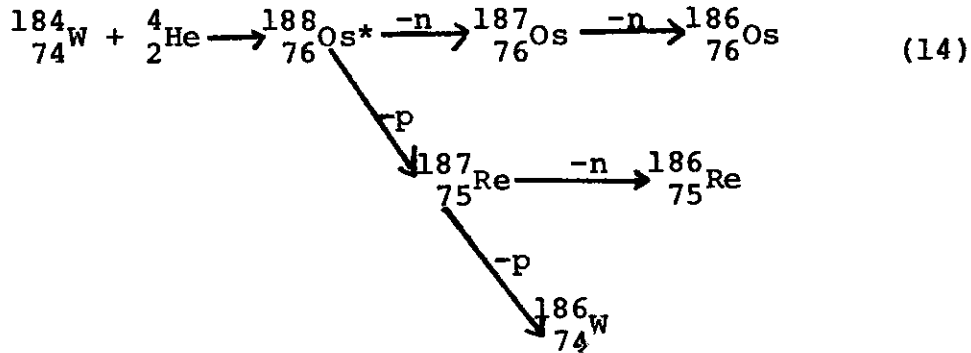
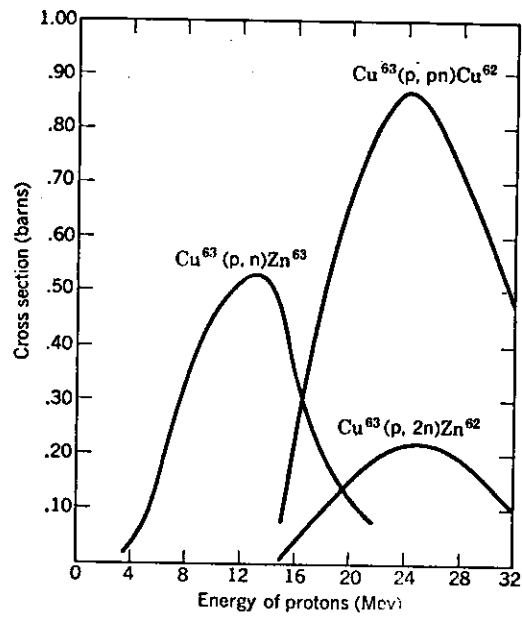


Figure 3 Sequence of events in the fission of a uranium nucleus by a neutron. In *a*, the neutron strikes the nucleus and is absorbed, causing the nucleus to undergo deformation, *b*. In about a hundredth of a trillionth of a second (10^{-14} second), one of the deformations, *c*, is so drastic that the nucleus cannot recover and fissions, *d*, releasing two or three neutrons. In about a trillionth of a second, the fission fragments have lost their kinetic energy and have come to rest, emitting a number of gamma rays. In the final stage, the excess nuclear energy is removed from the fission fragments by the emission of beta particles and gamma rays over a period of time from seconds to years. From "Nuclear Fission" by R. B. Leachman, copyright © 1965 by Scientific American, Inc. All rights reserved

E. Competing reactions.





At $E_p = 20$ MeV, ^{62}Cu , ^{62}Zn and ^{63}Zn all formed.

SECTION III

Reaction Cross Section

Cross section: probability of a nuclear reaction

Geometric cross sectional area of target nucleus

$$= \pi R^2 \text{ cm}^2 \quad (15)$$

assume $R = 1.4 \times 10^{-13} A^{1/3} \text{ cm}$, for $A = 200$:

$$\begin{aligned} \text{geometric cross section} &= (3.1416) (1.4 \times 10^{-13} \cdot 200^{1/3})^2 \\ &= 2.1 \times 10^{-24} \text{ cm}^2 \end{aligned} \quad (16)$$

1 barn (b) = 10^{-24} cm^2

A. Total cross section

Sum of all reactions, both scattering and absorption

$$\sigma_T = \underbrace{\sigma_{\text{elastic}} + \sigma_{\text{inelastic}}}_S + \sigma_{\text{spallation}} + \sigma_{\text{fission}} + \text{etc} \quad (18)$$

B. Partial cross sections

Cross section for each individual reaction

See Figures 2 and 4.

C. Measurement of cross section

1. Beam intensity (for charged particles) or flux (for neutrons)

Charged particles:

$$1 \mu\text{A} = \frac{6.28}{n} \times 10^{12} \text{ particles/sec.}$$

$$\mu\text{C} = \mu\text{A} \times t \text{ (in sec.)}$$

A containing way to

Ex: calculate the particles/s of a $^{14}\text{N}^{+4}$ beam striking a target if the Faraday cup reading gives 7500 μC collected during a bombardment of 2 h.

$$\# \mu A = 7500 \mu C \times \frac{1}{2h} \times \frac{1}{60m} \times \frac{1}{60s} = 1.04$$

$$\# \text{particles/s} = 1.04 \times \frac{6.28 \times 10^{12}}{4} = 1.63 \times 10^{12} \quad (14)$$

2. "Thin" target - beam passes through target with very small attenuation

$$\frac{dN_i}{dt} = \phi_0 n \sigma_i X = N_1 \phi \sigma \quad (15)$$

$\frac{dN_i}{dt}$ - number of reactions occurring in target per unit time.

ϕ_0 - number of particles incident upon target per unit time.

n - number of target nuclei per cm^3 of target.

σ_i - cross section for the production of product (in cm^2).

X - target thickness in cm.

N_1 - $n \cdot X$ - total number target nuclei/ cm^2 .

$$N_i = N_1 \sigma \phi t \quad (16)$$

If product is radioactive with decay constant λ ,

$$\frac{dN_i}{dt} = N_1 \sigma \phi - \lambda_1 N_i \quad (17)$$

$$A_i = N_1 \sigma \phi (1 - e^{-\lambda_1 t}) \quad (18)$$

N.B. Charged Particles: normally express N_1 in number target atoms/ cm^2 and ϕ in particles/s *(Beam is smaller than target)*

Neutrons: normally express N_1 in total number target atoms and ϕ in neutrons/ cm^2/s . *(Beam is larger than target)*

3. Thick target - beam attenuated or completely stopped in target.

The loss in intensity of flux through thickness N is:

$$-d\phi = \phi n \overline{\sigma}_i dN \quad (19)$$

Integrating we get:

$$\phi_x = \phi_0 e^{-n\overline{\sigma}_i x} \quad (20a)$$

or

$$\phi_0 - \phi_x = \phi_0 (1 - e^{-n\overline{\sigma}_i x}) = N_i \quad (20b)$$

ϕ_0 = initial intensity

ϕ_x = intensity at thickness x

SECTION IV

Reaction Mechanisms

A. Compound nucleus mechanism

1. Target and projectile fuse
2. Excitation energy equilibrated among nucleons of compound nucleus
3. Excitation energy removed by nucleon evaporation and decay path roughly independent of formation path

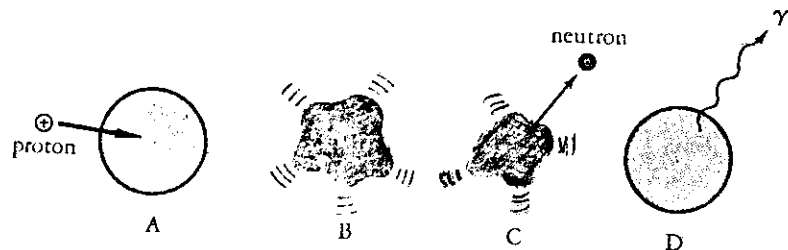


Figure 5. A simple representation of the formation of an excited compound nucleus B and its subsequent deexcitation by neutron evaporation C and gamma ray emission D.

B. Direct interaction mechanism

1. Projectile initially interacts with individual nucleons in target
2. After initial interaction, enters compound nucleus stage

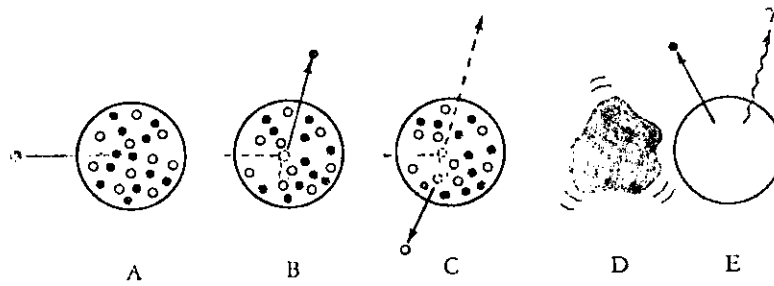
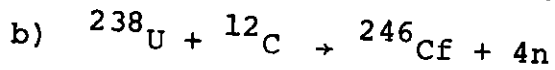
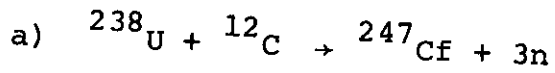


Figure 6. A simple representation of a high energy reaction in which a neutron (B) and a proton (C) are directly knocked out before formation of the compound nucleus D and subsequent deexcitation by evaporation and gamma ray emission E.

SECTION V

Problems

1. Calculate the energy thresholds for the following reactions:

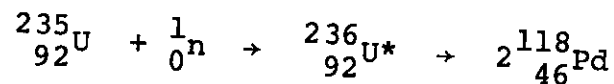


$$M_{{}^{238}\text{U}} = 238.1245; \quad M_{{}^{246}\text{Cf}} = 246.1451; \quad M_{{}^{247}\text{Cf}} = 247.1476;$$

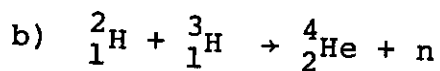
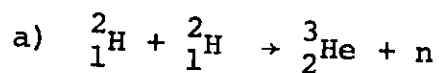
$$M_{{}^{12}\text{C}} = 12.0038.$$

2. Calculate the coulomb barrier for ${}^{238}_{92}\text{U} + {}^{12}_6\text{C}$ using $R_0 = 1.4 \times 10^{-13}$ cm. Compare this answer with that of Problem 1.

3. Use Fig. 1 of Part 1 to estimate the energy release in nuclear fission in the reaction



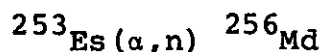
4. Calculate the energy released in the following thermonuclear reactions:



$$M_{{}^2\text{H}} = 2.0147; \quad M_{{}^3\text{H}} = 3.0170; \quad M_{{}^3\text{He}} = 3.0170; \quad M_{{}^4\text{He}} = 4.0038$$

5. How long must 25 g of cobalt be irradiated in a neutron flux of 10^{13} n/cm²/s in a reactor to produce 1000 Ci of ${}^{60}\text{Co}$? The $\sigma(n, \gamma)$ for ${}^{59}\text{Co}$ is 30.5 b and $t_{1/2}$ (${}^{60}\text{Co}$) is 5.2y; assume no attenuation of the flux.

6. In a thick target experiment, a 0.01/cm thickness of Cd causes a 69% reduction in intensity of a neutron beam passing through it. The density of Cd is 8.65g/cm^3 . What is the cross section for Cd to these neutrons?
7. In planning the discovery experiments of element 101, Md, it was known that a thin target of only 10^9 atoms of ^{253}Es was available. The cyclotron could provide a beam of $100\mu\text{A}$ and the estimated cross section was 1 mb for the reaction:



How long an irradiation time was required per atom of ^{256}Md (ignore its decay)?